1.4 Continuity and One-Sided Limits

- Determine continuity at a point and continuity on an open interval.
- Determine one-sided limits and continuity on a closed interval.
- Use properties of continuity.
- Understand and use the Intermediate Value Theorem.

Ex 1 What value of a will make f(x) continuous?

$$f(x) = \begin{cases} 3x^3, & x \le 1 \\ \overset{-2}{a}x + 5, & x > 1 \end{cases}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$$

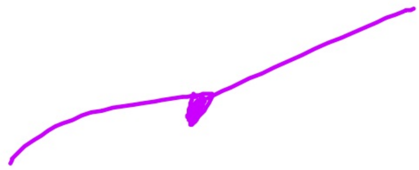
$$\lim_{x\to 1^-} 3x^3 = \lim_{x\to 1^+} ax+5 = 3$$

$$3 = a+5$$

$$\boxed{-2 = a}$$

Ex 2: For what value of c is g(x) continuous?

$$g(x) = \begin{cases} x^2 - c & x < 5 \\ 4x + 2c & x \geq 5 \end{cases}$$



$$\lim_{x \to 5^-} g(x) = \lim_{x \to 5^+} g(x) = g(5)$$

$$\lim_{x \to 5^-} (x^2 - c) = \lim_{x \to 5^+} (4x + 2c) = 25 - \frac{5}{3}$$

$$25 - c = 20 + 2c$$

$$5/3 = c$$

Ex 3

$$f(x) = \begin{cases} Ax - B & \text{if } x \le -1 \\ 2x^2 + 3Ax + B & \text{if } -1 < x \le 1 \\ 4 & \text{if } x > 1 \end{cases}$$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = f(-1)$$

$$\lim_{x \to -1^-} (Ax - B) = \lim_{x \to -1^+} (2x^2 + 3Ax + B)$$

$$-A - B = 2 - 3A + B$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$$\lim_{x \to 1^-} (2x^2 + 3Ax + B) = \lim_{x \to 1^+} 4$$

$$2 + 3A + B = 4$$

$$2A - 2B = 2$$

$$A - B = 1$$

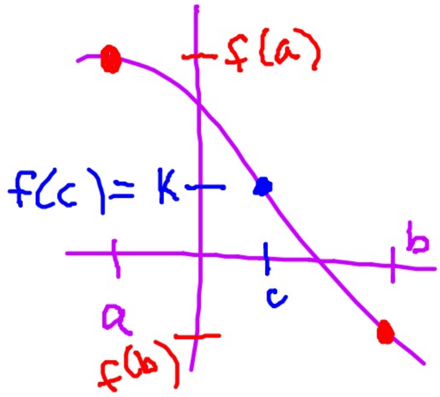
$$\begin{array}{r} 3A + B = 2 \\ + \quad A - B = 1 \\ \hline 4A = 3 \end{array}$$

$$A = \frac{3}{4} \qquad B = \frac{-1}{4}$$

THEOREM 1.13 INTERMEDIATE VALUE THEOREM

If f is continuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that

$$f(c) = k.$$



$a < c < b$

$f(b) < f(c) < f(a)$

Use IVT to prove that there will be a value c on the closed interval such that $\boxed{f(c) = 0}$

$$f(x) = x^3 + 5x - 3 \qquad [0, 1]$$

f(x) cont. [0, 1] ✓

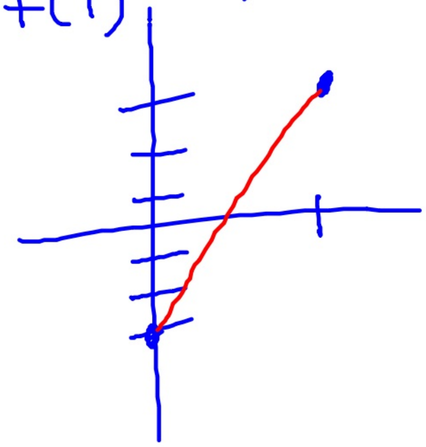
$0 < c < 1$

$f(0) = -3$

$f(a) < f(c) < f(b)$

$f(1) = 3$

$-3 < f(c) < 3$



Since f(x) is continuous on [0, 1] and f(0) < 0 < f(1), by IVT there exists a value c such that f(c) = 0

Prove that there is a value c guaranteed by IVT. Then, find c.

$$f(x) = \frac{x^2 + x}{x - 1}, \quad \left[\frac{5}{2}, 4\right], \quad f(c) = 6$$

$f(x)$ cont. on $\left[\frac{5}{2}, 4\right]$ ✓

Since f(x) is continuous on [2.5, 4], and f(2.5)<6<f(4), by IVT there exists a value c such that f(c) = 6

$f\left(\frac{5}{2}\right) = \frac{35}{6} = 5\frac{5}{6}$

$f(4) = \frac{20}{3} = 6\frac{2}{3}$

$$6 = \frac{x^2 + x}{x - 1}$$

$$6x - 6 = x^2 + x$$

$$0 = x^2 - 5x + 6$$

$$0 = (x-2)(x-3)$$

$x = \cancel{2}, 3$

2 is not in [2.5, 4]

$\boxed{c = 3}$