1.3 Evaluating Limits Analytically

- Evaluate a limit using properties of limits.
- Develop and use a strategy for finding limits.
- Evaluate a limit using dividing out and rationalizing techniques.
- Evaluate a limit using the Squeeze Theorem.

THEOREM 1.1 SOME BASIC LIMITS

Let b and c be real numbers and let n be a positive integer.

1. $\lim_{x \to c} b = b$ **2.** $\lim_{x \to c} x = c$ **3.** $\lim_{x \to c} x^n = c^n$

[T]HEOREM 1.2 PROPERTIES OF LIMITS

[L]et b and c be real numbers, let n be a positive integer, and let f and g be [fu]nctions with the following limits.

$$\lim_{x \to c} f(x) = L \qquad \text{and} \qquad \lim_{x \to c} g(x) = K$$

Scalar multiple: $\lim_{x \to c} [b f(x)] = bL$

Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$

Product: $\lim_{x \to c} [f(x)g(x)] = LK$

Quotient: $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{K}$, provided $K \neq 0$

Power: $\lim_{x \to c} [f(x)]^n = L^n$

Strategy for finding limits

1. Direct Substitution
2. Algebraic techniques (factoring or rationalizing)
3. Special Cases

THEOREM 1.9 TWO SPECIAL TRIGONOMETRIC LIMITS

1. $\lim_{x \to 0} \frac{\sin x}{x} = 1$ **2.** $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

Ex 1

$$\lim_{x\to 2}\frac{\sqrt{x+2}}{x-4}$$

$$\frac{\sqrt{2+2}}{2-4}$$

$$\frac{2}{-2}$$

$$-1$$

Ex 2

$$\lim_{x\to 7}\sec\left(\frac{\pi x}{6}\right)$$

$$\sec\frac{7\pi}{6}$$

$$-\frac{2}{\sqrt{3}}$$

Ex 3

$$\lim_{x \to 5\pi/3} \cos x = \frac{1}{2}$$

Ex 4

$$\lim_{x \to -1} \frac{2x^2 - x - 3}{x + 1}$$

$$\lim_{x \to -1} \frac{\cancel{(x+1)}(2x-3)}{\cancel{x+1}}$$

$$-5$$

Ex 5

$$\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$$

$$\lim_{x \to -1} \frac{\cancel{(x+1)}(x^2 - x + 1)}{\cancel{(x+1)}}$$

$$(-1)^2 - (-1) + 1$$

$$1 + 1 + 1$$

$$3$$

Ex 6

$$\lim_{x \to 3} \frac{\sqrt{x+1}-2}{x-3} \frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)}$$

$$\lim_{x \to 3} \frac{\cancel{x+1-4}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$$

$$\lim_{x \to 3} \frac{1}{\sqrt{x+1}+2}$$

$$\frac{1}{4}$$

Ex 7

$$\lim_{x\to 0} \frac{[1/(x+4)] - (1/4)}{x} = \lim_{x\to 0} \left(\frac{\frac{1}{x+4} - \frac{1}{4}}{\frac{x}{1}} \right) \frac{4(x+4)}{4(x+4)}$$

$$\lim_{x\to 0} \frac{4-(x+4)}{4x(x+4)}$$

$$\lim_{x\to 0} \frac{-\cancel{x}}{4\cancel{x}(x+4)}$$

$$-\frac{1}{16}$$

Ex 8

$$\lim_{x\to 0}\frac{3(1-\cos x)}{x} = 3\left[\lim_{x\to 0}\frac{(1-\cos x)}{x}\right] = 0$$

*graph $y=\frac{1-\cos x}{x}$

Ex 9

$$\lim_{t\to 0}\frac{\sin(3t)}{(2t)} = \frac{1}{2}\lim_{x\to 0}\frac{\sin(3x)}{x}\cdot\frac{3}{3}$$

$$= \frac{3}{2}\left[\lim_{x\to 0}\frac{\sin 3x}{3x}\right]$$

$$\frac{3}{2}$$

$$\lim_{x\to 0} \frac{\sin 4x}{11x}$$

$$\frac{1}{11}\lim_{x\to 0} \frac{\sin 4x}{x}\left(\cdot\frac{4}{4}\right)$$

$$\frac{4}{11}\left[\lim_{x\to 0} \frac{\sin 4x}{4x}\right]$$

$$\frac{4}{11}\cdot 1$$

$$\frac{4}{11}$$

Ex 10: Given f(x) = 5x - 2,

$f(x) = 5x - 2$

$f(x+\Delta x) =$

$5(x+\Delta x) - 2$

find $\lim\limits_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{\Delta x}$.

$$\lim_{\Delta x \to 0} \frac{5(x+\Delta x) - 2 - (5x - 2)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{5x} + 5\cancel{\Delta x} \cancel{- 2} \cancel{- 5x} \cancel{+ 2}}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0} 5 = 5$$

Ex. 11

$$\lim_{x \to c} f(x) = \tfrac{3}{2}$$

$$\lim_{x \to c} g(x) = \tfrac{1}{2}$$

(a) $\lim_{x \to c} [4f(x)]$

(b) $\lim_{x \to c} [f(x) + g(x)]$

(c) $\lim_{x \to c} [f(x)g(x)]$

(d) $\lim_{x \to c} \dfrac{f(x)}{g(x)}$