Analyzing Polynomial Functions Worksheet

I. Describe the end behavior of each function.

1. $f(x) = x^3 - 4x^2 + 7$
2. $f(x) = -x^2 + 4x$
3. $f(x) = -6x^5 - 4x^3 + 5x + 2$
4. $f(x) = 3x^2 - 6x + 11$

II. State the maximum number of turns the graph of each function could have.

5. $f(x) = x^5 - 4x^3 + 5x + 1$
6. $f(x) = -x^2 - 1$

III.
a) Using your graphing calculator sketch the function.
b) Determine if the zeros of each function has an even or odd multiplicity. Explain.

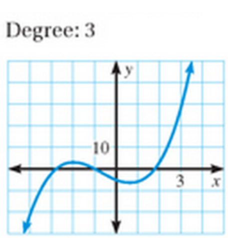
7. $f(x) = -x^2 - 6x - 7$
8. $f(x) = x^2 + 2$
9. $f(x) = x^3 - 3x - 2$

IV.
a) Discuss the end behavior.
b) How many turning points does the function have?
c) Find the real zero(s) on the calculator. Round to the nearest thousandth.
d) Determine the point(s) at which the function has a relative maximum/minimum – label each as a max/min. Round to the nearest thousandth.
e) Determine the domain on which the function is increasing/decreasing. State your answer in *INTERVAL* notation. Round to the nearest thousandth.
f) Determine the domain on which the function is positive/negative. State your answer in *INTERVAL* notation. Round to the nearest thousandth.
g) State the domain and range in *SET* notation. Round to the nearest thousandth.
h) Determine if f(x) is even, odd, or neither. *Explain*.

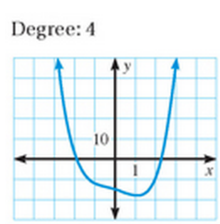
10. $f(x) = -x^3 + 2x^2 - 4$
11. $f(x) = x^4 + 2x - 1$
12. $f(x) = -x^4 + 4x^2 + 1$

V. Determine the numbers of positive real zeros, negative real zeros, and imaginary zeros for the function with the given degree and graph. *Explain* your reasoning.

13.



14.



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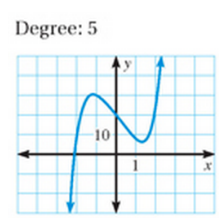

ANSWERS

1.
$x \to -\infty, f(x) \to -\infty$
$x \to \infty, f(x) \to \infty$

2.
$x \to -\infty, f(x) \to -\infty$
$x \to \infty, f(x) \to -\infty$

3.
$x \to -\infty, f(x) \to \infty$
$x \to \infty, f(x) \to -\infty$

4.
$x \to -\infty, f(x) \to \infty$
$x \to \infty, f(x) \to \infty$

5. 4
6. 1

7.
   a. See your graphing calc.
   b. The graph has a cross at both zeros therefore both zeros have an *odd* multiplicity.

8.
   a. See your graphing calc.
   b. There are no real zeros, therefore both zeros must be imaginary. Since imaginary zeros come in pairs, each zero has an *odd* multiplicity.

9.
   a. See your graphing calc.
   b. The graph has a cross at x=-1, therefore this zero has an *odd* multiplicity. The graph has a bounce at x=2, therefore this zero has an *even* multiplicity.

10.
a) As $x \to -\infty$, $f(x) \to \infty$; As $x \to \infty$, $f(x) \to -\infty$
b) 2
c) $x \approx -1.130$
d) Relative Maximum: $(1.333,\ -2.815)$, Relative Minimum: $(0,\ -4)$
e) Increasing: $(0,1.333)$, Decreasing: $(-\infty,0)\cup(1.333,\infty)$
f) Positive: $(-\infty,-1.130)$, Negative: $(-1.130,\infty)$
g) Domain: $\{x|x \in R\}$, Range: $\{y|y \in R\}$
h) $f(x)$ is neither even nor odd since the graph does not have y-axis or origin symmetry.

11.
a) As $x \to -\infty$, $f(x) \to \infty$; As $x \to \infty$, $f(x) \to \infty$
b) 1
c) $x \approx -1.395,\ 0.475$
d) Relative Maximum: none, Relative Minimum: $(-0.794,\ -2.191)$
e) Increasing: $(-0.494,\infty)$, Decreasing: $(-\infty,-0.494)$
f) Positive: $(-\infty,-1.395)\cup(0.475,\infty)$, Negative: $(-1.395,\ 0.475)$
g) Domain: $\{x|x \in R\}$, Range: $\{y|y \geq -2.191\}$
h) $f(x)$ is neither even nor odd since the graph does not have y-axis or origin symmetry.

12.

a) As $x \to -\infty$, $f(x) \to -\infty$; As $x \to \infty$, $f(x) \to -\infty$

b) 3

c) $x = -2.058,\ 2.058$

d) Relative Maximum: $(\pm 1.414,\ 5)$, Relative Minimum: $(0,\ 1)$

e) Increasing: $(-\infty,-1.414) \cup (0,1.414)$, Decreasing: $(-1.414,0) \cup (1.414,\infty)$

f) Positive: $(-2.058,\ 2.058)$, Negative: $(-\infty,-2.058) \cup (2.058,\infty)$

g) Domain: $\{x | x \in R\}$, Range: $\{y | y \le 5\}$

h) $f(x)$ is an even function since its graph has y-axis symmetry.

13. 1 positive real zero, 2 negative real zero, 0 imaginary zeros

14. 1 positive real zero, 1 negative real zero, 2 imaginary zeros

15. 0 positive real zero, 1 negative real zero, 4 imaginary zeros