(Mostly) Everything you should know for AP Calculus...

1. Limit Definition of the Derivative: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

2. Limit Definition of the Derivative (Alternative Form): $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

3. Average Rate of change of $f(x)$ on $[a,b]$: $\frac{f(b)-f(a)}{b-a}$

4. Average Value of $f(x)$ on $[a,b]$: $\frac{1}{b-a}\int_a^b f(x)\,dx$

5. Intermediate Value Theorem:
   - Conditions: f(x) is continuous on the closed interval, [a, b]
   - Conclusion: There is a value c such that $f(a) \le f(c) \le f(b)$ or $f(b) \le f(c) \le f(a)$ and $a \le c \le b$.

6. Rolle's Theorem:
   - Conditions: f(x) is continuous on the closed interval, [a, b], and differentiable on the open interval (a, b) and f(a)=f(b)
   - Conclusion: $f'(c) = 0$ and a<c<b

7. Mean Value Theorem:
   - Conditions: f(x) is continuous on the closed interval, [a, b], and differentiable on the open interval (a, b)
   - Conclusion: $f'(c) = \frac{f(b)-f(a)}{b-a}$ and a<c<b

8. Extreme Value Theorem:
   - Conditions: f(x) is continuous on the closed interval, [a, b]
   - Conclusion: f(x) has an absolute maximum and absolute minimum at a critical number or an endpoint on [a, b]

9. Double Angle Identities:
   - $\sin 2x = 2\sin x\cos x$
   - $\cos 2x = \cos^2 x - \sin^2 x$

10. Power Reducing Identities:
    - $\sin^2 x = \frac{1-\cos 2x}{2}$
    - $\cos^2 x = \frac{1+\cos 2x}{2}$

11. Critical Number: *f* has a critical number when $f' = 0$ or is undefined

12. Increasing/Decreasing:
    - *f* is increasing when $f' > 0$
    - *f* is decreasing when $f' < 0$

13. Concavity:
    - *f* is concave up when f' is increasing and $f'' > 0$
    - *f* is concave down when f' is decreasing and $f'' < 0$

14. Relative Extrema (1^{st} Derivative Test):
    - *f* has a relative maximum when f' changes from positive to negative.
    - *f* has a relative minimum when f' changes from negative to positive.

15. Relative Extrema (2nd Derivative Test):
   - *f* has a relative maximum when $f' = 0$ or is undefined and $f'' < 0$.
   - *f* has a relative minimum when $f' = 0$ or is undefined and $f'' > 0$.
16. Point of Inflection
   - *f* has a point of inflection when f' has relative extrema and f'' changes signs.
17. Fundamental theorem of calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$
   - $\int_a^b f(x)dx$ is the area under the curve of f(x)
   - $\int_b^a f(x)dx$ is negative if the area is below the x-axis
18. Accumulation Functions: $\int_c^{g(x)} f(t)\,dt$
   - To find the derivative: $\frac{d}{dx}\left[\int_c^{g(x)} f(t)\,dt\right] = f\left(g(x)\right)g'(x)$ (2ND FTC)
19. Volume by discs (horizontal axis): $\pi\int_a^b r^2\,dx$
20. Volume by discs (vertical axis): $\pi\int_a^b r^2\,dy$
21. Volume by washers (horizontal axis): $\pi\int_a^b \left(R^2 - r^2\right)dx$
22. Volume by washers (vertical axis): $\pi\int_a^b \left(R^2 - r^2\right)dy$
23. Volume by cross sections perpendicular to the x-axis: $\int_a^b A(x)\,dx$
24. Volume by cross sections perpendicular to the y-axis: $\int_a^b A(y)\,dy$
25. Position/ Velocity/Acceleration (AB):
   - Speed is increasing when: acceleration and velocity have the same signs
   - Speed is decreasing when: acceleration and velocity have opposite signs
26. Given a graph of *f* and $g(x) = \int_0^x f(t)\,dt$:
   - The graph *f* is the graph of g'
   - $\int_0^x f(t)dt$ is the AREA under the curve $f(t)$.
   - To evaluate *g*(x), evaluate the integral by using geometric shapes.
27. Derivative Approximations

x	f(x)
a	e
b	f
d	g

To approximate $f'(c) \approx \frac{f(d) - f(b)}{d - b}$

28. Tangent Line Approximations

1. Write the tangent line at the given point: $(a, f(a))$

$$y - f(a) = f'(a)(x-a)$$

2. Then plug in the point $x = x_1$ and solve for y.

$$y = f'(a)(x_1 - a) + f(a)$$

29. Absolute extrema – Compare the y-values of the relative extrema AND the endpoints. If there is only 1 critical number then the critical number is both a relative and absolute extrema.

30. Particle Motion - Position/ Velocity/ Acceleration
- PVAJ:
  - Position: $x(t)$
  - Velocity: $x'(t) = v(t)$
  - Acceleration: $x''(t) = v'(t) = a(t)$
- SPEED
  - Speed: $|v(t)|$
  - INCREASING – velocity and acceleration have the same signs
  - DECREASING – velocity and acceleration have opposite signs
- Initially: t=0
- At Rest: v(t)=0
- Particle Moving Right: v(t)>0
- Particle Moving Left: v(t)<0
- Total Distance on [a, b]: $\int_a^b |v(t)|\,dt$
- Average velocity on [a, b]: $\dfrac{x(b)-x(a)}{b-a}$ or $\dfrac{1}{b-a}\int_a^b v(t)\,dt$
- Instantaneous velocity at t=a: $v(a) = x'(a)$

31. Derivative Formulas

$$\frac{d}{dx}[c]=0 \qquad \frac{d}{dx}[x]=1 \qquad \frac{d}{dx}[cx]=c \qquad \frac{d}{dx}\left[x^c\right]=cx^{c-1}$$

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x)+f'(x)g(x) \qquad \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x) \qquad \frac{d}{dx}[\ln x] = \frac{1}{x} \qquad \frac{d}{dx}\left[e^x\right]=e^x \qquad \frac{d}{dx}[\sin x]=\cos x$$

$$\frac{d}{dx}[\cos x]=-\sin x \qquad \frac{d}{dx}[\tan x]=\sec^2 x \qquad \frac{d}{dx}[\cot x]=-\csc^2 x \qquad \frac{d}{dx}[\sec x]=\sec x\tan x$$

$$\frac{d}{dx}[\csc x]=-\csc x\cot x \qquad \frac{d}{dx}[\arcsin x]=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\arctan x]=\frac{1}{1+x^2}$$

32. Integration Formulas

$$\int dx = x+c \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1}+c \qquad \int \frac{dx}{x} = \ln|x|+c$$

$$\int e^x\,dx = e^x+c \qquad \int \sin x\,dx = -\cos x + c \qquad \int \cos x\,dx = \sin x + c$$

$$\int \tan x\,dx = -\ln|\cos x|+c \qquad \int \csc x\,dx = -\ln|\csc x+\cot x|+c \qquad \int \sec x\,dx = \ln|\sec x+\tan x|+c$$

$$\int \cot x\,dx = \ln|\sin x|+c \qquad \int \sec^2 x\,dx = \tan x \qquad \int \csc^2 x\,dx = -\cot x + c$$

$$\int \sec x\tan x\,dx = \sec x + c \qquad \int \csc x\cot x\,dx = -\csc x + c \qquad \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a}+c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a}+c \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a}+c$$

33. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

34. Integration by parts: $\int u\,dv = uv - \int v\,du$

35. Arc Length of f on [a, b]: $\int_a^b \sqrt{1+\left(f'(x)\right)^2}\,dx$

36. Vectors

- Position: $\left(x(t), y(t)\right)$
- Velocity: $\left(x'(t), y'(t)\right)$
- Acceleration: $\left(x''(t), y''(t)\right)$
- Speed (or magnitude of the velocity vector): $\sqrt{\left(x'(t)\right)^2+\left(y'(t)\right)^2}$
- Distance traveled on [a, b]: $\int_a^b \sqrt{\left(x'(t)\right)^2+\left(y'(t)\right)^2}\,dt$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \frac{d^2y}{dx^2} = \frac{\frac{d}{dx}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

37. Polar

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

- Slope of a polar curve: $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$
- Area enclosed by a polar curve on $[\alpha, \beta]$: $\frac{1}{2}\int_\alpha^\beta r^2\,d\theta$
- Area between two polar curves on $[\alpha, \beta]$: $\frac{1}{2}\int_\alpha^\beta \left(R^2 - r^2\right)d\theta$
- Polar Arc Length: $\int_\alpha^\beta \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$

38. Basic 5 Maclaurin Series

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} \qquad \sin x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!} \qquad \cos x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad \arctan x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}$$

39. Lagrange Error Bound: $\left|\frac{f^{n+1}(z)(x-c)^{n+1}}{(n+1)!}\right|$

40. Alternating Series Error Bound: $\left|\text{1st neglected term}\right|$