(Mostly) Everything you should know for AP Calculus...

1. Limit Definition of the Derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

2. Limit Definition of the Derivative (Alternative Form): $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

3. Average Rate of change of $f(x)$ on $[a,b]$: $\frac{f(b) - f(a)}{b - a}$

4. Average Value of $f(x)$ on $[a,b]$: $\frac{1}{b-a} \int_a^b f(x)\,dx$

5. Intermediate Value Theorem:
   - Conditions: f(x) is continuous on the closed interval, [a, b]
   - Conclusion: There is a value c such that $f(a) \le f(c) \le f(b)$ or $f(b) \le f(c) \le f(a)$ and $a \le c \le b$.

6. Rolle's Theorem:
   - Conditions: f(x) is continuous on the closed interval, [a, b], and differentiable on the open interval (a, b) and f(a)=f(b)
   - Conclusion: $f'(c) = 0$ and a<c<b

7. Mean Value Theorem:
   - Conditions: f(x) is continuous on the closed interval, [a, b], and differentiable on the open interval (a, b)
   - Conclusion: $f'(c) = \frac{f(b) - f(a)}{b - a}$ and a<c<b

8. Extreme Value Theorem:
   - Conditions: f(x) is continuous on the closed interval, [a, b]
   - Conclusion: f(x) has an absolute maximum and absolute minimum at a critical number or an endpoint on [a, b]

9. Double Angle Identities:
   - $\sin 2x = 2 \sin x \cos x$
   - $\cos 2x = \cos^2 x - \sin^2 x$

10. Power Reducing Identities:
    - $\sin^2 x = \frac{1 - \cos 2x}{2}$
    - $\cos^2 x = \frac{1 + \cos 2x}{2}$

11. Critical Number: *f* has a critical number when $f' = 0$ or is undefined

12. Increasing/Decreasing:
    - *f* is increasing when $f' > 0$
    - *f* is decreasing when $f' < 0$

13. Concavity:
    - *f* is concave up when f' is increasing and $f'' > 0$
    - *f* is concave down when f' is decreasing and $f'' < 0$

14. Relative Extrema (1st Derivative Test):
    - *f* has a relative maximum when f' changes from positive to negative.
    - *f* has a relative minimum when f' changes from negative to positive.

15. Relative Extrema (2^{nd} Derivative Test):

- *f* has a relative maximum when $f' = 0$ or is undefined and $f'' < 0$.
- *f* has a relative minimum when $f' = 0$ or is undefined and $f'' > 0$.

16. Point of Inflection

- *f* has a point of inflection when f' has relative extrema and f'' changes signs.

17. Fundamental theorem of calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$

- $\int_a^b f(x)dx$ is the area under the curve of f(x)
- $\int_b^a f(x)dx$ is negative if the area is below the x-axis

18. Accumulation Functions: $\int_c^{g(x)} f(t)\,dt$

- To find the derivative: $\frac{d}{dx}\left[\int_c^{g(x)} f(t)\,dt\right] = f(g(x))g'(x)$ (2^{ND} FTC)

19. Volume by discs (horizontal axis): $\pi\int_a^b r^2\,dx$

20. Volume by discs (vertical axis): $\pi\int_a^b r^2\,dy$

21. Volume by washers (horizontal axis): $\pi\int_a^b \left(R^2 - r^2\right)dx$

22. Volume by washers (vertical axis): $\pi\int_a^b \left(R^2 - r^2\right)dy$

23. Volume by cross sections perpendicular to the x-axis: $\int_a^b A(x)\,dx$

24. Volume by cross sections perpendicular to the y-axis: $\int_a^b A(y)\,dy$

25. Position/ Velocity/Acceleration (AB):

- Speed is increasing when: acceleration and velocity have the same signs
- Speed is decreasing when: acceleration and velocity have opposite signs

26. Given a graph of *f* and $g(x) = \int_0^x f(t)\,dt$:

- The graph *f* is the graph of g'
- $\int_0^x f(t)dt$ is the AREA under the curve $f(t)$.
- To evaluate *g*(x), evaluate the integral by using geometric shapes.

27. Derivative Approximations

x	f(x)
a	e
b	f
d	g

To approximate $f'(c) \approx \dfrac{f(d) - f(b)}{d - b}$

28. Tangent Line Approximations

1. Write the tangent line at the given point: $(a, f(a))$

$$y - f(a) = f'(a)(x-a)$$

2. Then plug in the point $x = x_1$ and solve for y.

$$y = f'(a)(x_1 - a) + f(a)$$

29. Absolute extrema – Compare the y-values of the relative extrema AND the endpoints. If there is only 1 critical number then the critical number is both a relative and absolute extrema.

30. Particle Motion - Position/ Velocity/ Acceleration
    - PVAJ:
        - Position: $x(t)$
        - Velocity: $x'(t) = v(t)$
        - Acceleration: $x''(t) = v'(t) = a(t)$
    - SPEED
        - Speed: $|v(t)|$
        - INCREASING – velocity and acceleration have the same signs
        - DECREASING – velocity and acceleration have opposite signs
    - Initially: t=0
    - At Rest: v(t)=0
    - Particle Moving Right: v(t)>0
    - Particle Moving Left: v(t)<0
    - Total Distance on [a, b]: $\int_a^b |v(t)|\,dt$
    - Average velocity on [a, b]: $\dfrac{x(b)-x(a)}{b-a}$ or $\dfrac{1}{b-a}\int_a^b v(t)\,dt$
    - Instantaneous velocity at t=a: $v(a) = x'(a)$

31. Derivative Formulas

$\frac{d}{dx}[c] = 0$ $\quad \frac{d}{dx}[x] = 1$ $\quad \frac{d}{dx}[cx] = c$ $\quad \frac{d}{dx}[x^c] = cx^{c-1}$

$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$ $\quad \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$ $\quad \frac{d}{dx}[\ln x] = \frac{1}{x}$ $\quad \frac{d}{dx}[e^x] = e^x$ $\quad \frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[\cos x] = -\sin x$ $\quad \frac{d}{dx}[\tan x] = \sec^2 x$ $\quad \frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad \frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$ $\quad \frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$

32. Integration Formulas

$\int dx = x + c$ $\quad \int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ $\quad \int \frac{dx}{x} = \ln|x| + c$

$\int e^x\,dx = e^x + c$ $\quad \int \sin x\,dx = -\cos x + c$ $\quad \int \cos x\,dx = \sin x + c$

$\int \tan x\,dx = -\ln|\cos x| + c$ $\quad \int \csc x\,dx = -\ln|\csc x + \cot x| + c$ $\quad \int \sec x\,dx = \ln|\sec x + \tan x| + c$

$\int \cot x\,dx = \ln|\sin x| + c$ $\quad \int \sec^2 x\,dx = \tan x$ $\quad \int \csc^2 x\,dx = -\cot x + c$

$\int \sec x \tan x\,dx = \sec x + c$ $\quad \int \csc x \cot x\,dx = -\csc x + c$ $\quad \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$ $\quad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$