89.) $f(x) = 2x\sqrt{x-6}$

$(f^{-1})'(40) = \frac{1}{9}$

$40 = 2x\sqrt{x-6}$

$f^{-1}: (40, \ )$

$f: (\ , 40)$

reciprocal

$10 = x$

$f'(10) = 9$

57.) $f(x) = \arccos x$

$f'(x) = \frac{-1}{\sqrt{1-x^2}}$

$+2 = \frac{\pm 1}{\sqrt{1-x^2}}$

$2\sqrt{1-x^2} = 1$

$\sqrt{1-x^2} = \frac{1}{2}$

$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{6}\right)\left(-\frac{\sqrt{3}}{2}, \frac{5\pi}{6}\right)$

$1 - x^2 = \frac{1}{4}$

$\frac{3}{4} = x^2$

$\pm\frac{\sqrt{3}}{2} = x$

$y - \frac{\pi}{6} = -2\left(x - \frac{\sqrt{3}}{2}\right)$

$y - \frac{5\pi}{6} = -2\left(x + \frac{\sqrt{3}}{2}\right)$

54.) $y = \operatorname{arcsec} 4x$ $\left(\frac{\sqrt{2}}{4}, \frac{\pi}{4}\right)$

$$y' = \frac{4}{|4x|\sqrt{16x^2 - 1}}$$

$$y - \frac{\pi}{4} = \frac{4}{\sqrt{2}}\left(x - \frac{\sqrt{2}}{4}\right)$$

$$y' = \frac{1}{|x|\sqrt{16x^2 - 1}}$$

$$y'\left(\frac{\sqrt{2}}{4}\right) = \frac{1}{\frac{\sqrt{2}}{4}\sqrt{16 \cdot \frac{2}{16} - 1}} = \frac{1}{\frac{\sqrt{2}}{4}} = \frac{4}{\sqrt{2}}$$

25.) $g(x) = 3\arccos\frac{x}{2}$

$$g'(x) = 3\left(\frac{-\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}}\right)$$

$$= \frac{-\cancel{\frac{3}{2}}}{\sqrt{\frac{4-x^2}{\cancel{4}}}} \qquad \frac{-\frac{3}{2}}{\frac{1}{2}} = -\frac{3}{\cancel{2}} \cdot \frac{\cancel{2}}{1}$$

$$= \frac{-3}{\sqrt{4-x^2}}$$

41.) $g(t) = \tan(\arcsin t)$

$$g(t) = \frac{t}{\sqrt{1-t^2}}$$

$$\tan\theta = \frac{t}{\sqrt{1-t^2}}$$

$$g'(t) = \frac{(1-t^2)^{1/2}\cdot 1 + t\cdot\frac{1}{\cancel{2}}(1-t^2)^{-1/2}(-\cancel{2}t)}{1-t^2}$$

$$= \frac{\frac{\sqrt{1-t^2}}{1} + \frac{t^2}{\sqrt{1-t^2}}}{1-t^2} = \frac{\frac{1-\cancel{t^2}+\cancel{t^2}}{\sqrt{1-t^2}}}{(1-t^2)^1} = \frac{1}{\sqrt{1-t^2}}\cdot\frac{1}{1-t^2}$$

$$= \frac{1}{(1-t^2)^{3/2}}$$

Rates of change and Rectilinear Motion

- Application of the Derivative: *maximize,minimize values*

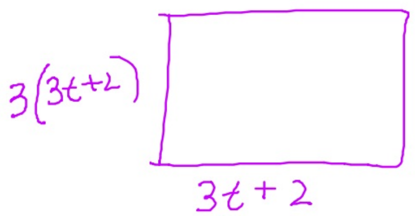
where is a particle moving to the left (right); slope, rate of change

- Rate of Change(definition): *Change in y divided by change in x*

(slope)

Examples:

- ____________________________
- ____________________________
- ____________________________
- ____________________________
- ____________________________



$$A = 3(3t+2)^2$$

$$A' = (6)(3t+2)^1 \cdot 3$$

$$A' = 18(3t+2)$$

$$A'(1) = 18 \cdot 5 = 90 \text{ m/hr}$$

ex 2

$P'(2) = 31.55$ bacteria/hr

When you are asked to find the rate of change, you are finding the slope (derivative).

ex:

The temperature, in degrees Fahrenheit $(^\circ F)$, of water in a pond is modeled by the function H given by $H(t) = 55 - 9\cos\left(\frac{2\pi}{365}(t+10)\right)$, where t is the number of days since January 1 $(t = 0)$. What is the instantaneous rate of change of the temperature of the water at time $t = 90$ days?

(A) 0.114°F/day

(B) 0.153°F/day

(C) 50.252°F/day

(D) 56.350°F/day

- Rectilinear Motion Problems – When we talk about these types of problems, we often talk about three types of functions

1. Position:

   Notation: $x(t)$, $s(t)$

   Application(s): where is the particle at $t = 2$; what is the max distance from the origin?

2. Velocity : change in distance over change in time

   Notation: $s'(t)$, $x'(t)$, $v(t)$ m|sec

   Application(s): what is the velocity at $t = 2$; when is the velocity 5 m/sec

3. Acceleration change in velocity over change in time

   Notation: $s''(t)$, $x''(t)$, $v'(t)$, $a(t)$ m|sec^2

   Application(s): what is the acceleration at $t = 3$? What is the acceleration

Ex 3: At time t=0 seconds a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$.

a) When does the diver hit the water?

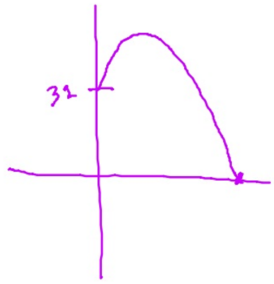
$s(t) = 0$

$0 = -16t^2 + 16t + 32$

$0 = -16(t^2 - t - 2) = -16(t-2)(t+1)$

$\cancel{t = -1}, 2$

$t = 2\text{sec}$



b) What is the diver's velocity at impact?

$v(t) = -32t + 16$

$v(2) = -48$ ft/sec

- Average Velocity vs. Instantaneous Velocity
  - Average Velocity: *change in distance over change in time*

    (You must be given an interval)

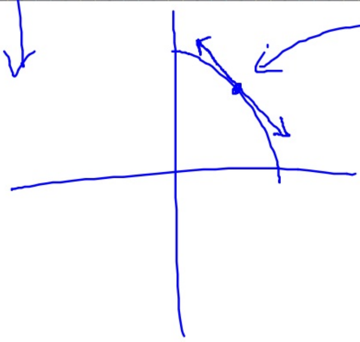
    Formula: $\frac{s(b)-s(a)}{b-a}$ on $[a, b]$
  - Instantaneous Velocity:

    the velocity at a point in time

    Formula: $s'(a)$

no calculus required

just slope of secant



slope of tangent

- Speed: $|v(t)|$

- Rest: the velocity is equal to 0

- Left and Right Motion:
  - Left: where $v(t) < 0$
  - Right: where $v(t) > 0$
  - Changes Direction: when $v(t)$ changes from positive to negative or negative to positive (changes signs)

Ex 4: A billiard ball is dropped from a height of 100 ft, its height *s* at time *t* is given by the position function $s(t) = -16t^2 + 100$, where *s* is measured in feet and *t* is measured in seconds.

a) Find the average velocity over time interval [1, 2].

$(1, 84)(2, 36)$

$$\frac{s(2)-s(1)}{2-1} = \frac{36-84}{2-1} = -48 \text{ ft/sec}$$

b) Find the Instantaneous velocity at the endpoints of the interval.

$$s'(t) = -32t$$

$s'(1) = -32$ ft/sec $\quad s'(2) = -64$ ft/sec

c) Find the speed at the endpoints of the interval.

$$|v(1)| = 32 \text{ ft/sec}$$

$$|v(2)| = 64 \text{ ft/sec}$$

Ex 5: A particle starts at time t=0 and moves along the x-axis so that its position at any time $t \ge 0$ is given by $x(t) = (t-1)^3(2t-3)$.

a) Find the velocity of the particle at any time $t \ge 0$. Simplify.

$$x'(t) = (t-1)^2(8t-11)$$

b) Determine the values of t for which the particle is at rest.

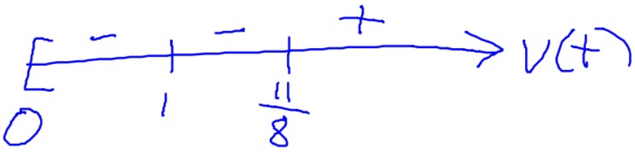
$$0 = (t-1)^2(8t-11)$$

$$t = 1, 11/8$$

At rest at t = 1, 11/8 because v(t) = 0 at these times.

c) Determine the values of t for which the particle is moving to the left. JYA.

$$x'(t) = (t-1)^2(8t-11)$$



The particle is moving to the left on (0,1) U (1,11/8) because v(t) < 0 on these intervals.

d) Determine the values of t for which the particle is moving to the right. Justify your answer.

Moving to the right on (11/8,inf) because v(t) > 0 on this interval.

e) Determine the values of t for which the particle changes direction. Justify your answer.

t = 11/8 because v(t) changes signs at this time OR
v(t) changes from negative to positive at this time

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A particle moves along the x-axis so that at time t its position is given by $x(t) = \sin\left(\pi t^2\right)$ for $-1 \le t \le 1$.

(a) Find the velocity at time t.

(b) Find the acceleration at time t.

(c) For what values of t does the particle change direction?

(d) Find all values of t for which the particle is moving to the left.

Derivatives of inverse functions

Derivatives of inverse trig functions

Motion on a line