L^Hopital's Rule

Some limits can be evaluated using L^Hopital's Rule:

Indeterminate Forms: $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, $\infty - \infty$, 0^0, 1^∞, ∞^0

L'Hopital's Rule. Suppose that $f(x)$ and $g(x)$ are differential functions and $g'(x) \neq 0$ on an open interval I that contains a (except possibly for a). If

$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = 0$$

or

$$\lim_{x\to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x\to a} g(x) = \pm\infty,$$

then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

provided the right-hand-limit exists (or equals $\pm\infty$).

Note: Our goal will always be to write any indeterminate forms as one of the first two: $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

Example of L^HOPITAL'S RULE: INDETERMINATE FORM $\frac{0}{0}$ When you try and evaluate the limit, the result is $\frac{0}{0}$. So take the derivative of the numerator and denominator then evaluate.

***WARNING: YOU CANNOT USE L^HOPITAL'S FOR ALL LIMITS, JUST LIMITS IN INDETERMINATE FORM. THE ONLY INDETERMINATE FORMS ON THE AP EXAM ARE $\frac{0}{0}$ *and* $\frac{\infty}{\infty}$

***SOME OF THESE LIMITS CAN BE EVALUATED DIFFERENTLY. FOR INSTANCE, EXAMPLE 1 CAN BE FACTORED, EXAMPLE 2 IS A SPECIAL CASE, AND EXAMPLE 3 IS END BEHAVIOR.

Examples:

1) $\lim_{x\to 4} \frac{x^2-16}{x^2-5x+4} =$ L'HOP $\lim_{x\to 4} \frac{2x}{2x-5} = \frac{8}{3}$	2) $\lim_{x\to 0} \frac{\sin x}{x} =$ LHop $\to$ $\lim_{x\to 0} \frac{\cos x}{1} = 1$	3) $\lim_{x\to\infty} \frac{5+3x^2}{4x^2+1} =$ LHop $\lim_{x\to\infty} \frac{6x}{8x} = \frac{6}{8} = \frac{3}{4}$
4) $\lim_{x\to 0} \frac{e^x - \cos x - 2x}{x^2-2x} =$ LHop $\to$ $\lim_{x\to 0} \frac{e^x + \sin x - 2}{2x-2} = \frac{-1}{-2} = \frac{1}{2}$	5) $\lim_{x\to 2} \frac{x^2-4x+4}{x^3-12x+16} =$ LHop $\lim_{x\to 2} \frac{2x-4}{3x^2-12}$ LHop $\to$ $\lim_{x\to 2} \frac{2}{6x} = \frac{1}{6}$	6) $\lim_{x\to\pi/2} \frac{1-\sin x}{1+\cos 2x} =$ LHop $\lim_{x\to\frac{\pi}{2}} \frac{+\cos x}{+2\sin 2x} = \lim_{x\to\frac{\pi}{2}} \frac{-\sin x}{4\cos 2x} = \frac{-1}{-4}$ $1/4$