

1) Calculator

For  $0 \leq t \leq 6$ , a particle is moving along the  $x$ -axis. The particle's position,  $x(t)$ , is not explicitly given. The velocity of the particle is given by  $v(t) = 2\sin(e^{t/4}) + 1$ . The acceleration of the particle is given by  $a(t) = \frac{1}{2}e^{t/4}\cos(e^{t/4})$  and  $x(0) = 2$ .

- (a) Is the speed of the particle increasing or decreasing at time  $t = 5.5$ ? Give a reason for your answer.
- (b) Find the average velocity of the particle for the time period  $0 \leq t \leq 6$ .
- (c) Find the total distance traveled by the particle from time  $t = 0$  to  $t = 6$ .
- (d) For  $0 \leq t \leq 6$ , the particle changes direction exactly once. Find the position of the particle at that time.

## 2) Calculator

|                             |    |    |    |    |    |
|-----------------------------|----|----|----|----|----|
| $t$<br>(minutes)            | 0  | 2  | 5  | 9  | 10 |
| $H(t)$<br>(degrees Celsius) | 66 | 60 | 52 | 44 | 43 |

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function  $H$  for  $0 \leq t \leq 10$ , where time  $t$  is measured in minutes and temperature  $H(t)$  is measured in degrees Celsius. Values of  $H(t)$  at selected values of time  $t$  are shown in the table above.

- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time  $t = 3.5$ . Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of  $\frac{1}{10} \int_0^{10} H(t) dt$  in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate  $\frac{1}{10} \int_0^{10} H(t) dt$ .
- (c) Evaluate  $\int_0^{10} H'(t) dt$ . Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time  $t = 0$ , biscuits with temperature  $100^\circ\text{C}$  were removed from an oven. The temperature of the biscuits at time  $t$  is modeled by a differentiable function  $B$  for which it is known that  $B'(t) = -13.84e^{-0.173t}$ . Using the given models, at time  $t = 10$ , how much cooler are the biscuits than the tea?

### 3) No Calculator

Let  $f$  be a function defined by  $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

- (a) Show that  $f$  is continuous at  $x = 0$ .
- (b) For  $x \neq 0$ , express  $f'(x)$  as a piecewise-defined function. Find the value of  $x$  for which  $f'(x) = -3$ .
- (c) Find the average value of  $f$  on the interval  $[-1, 1]$ .

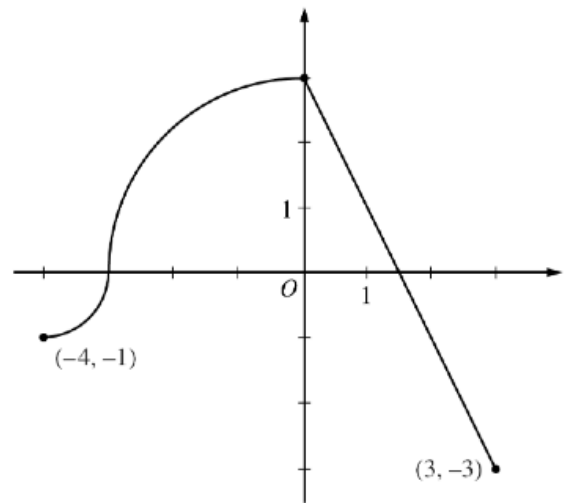
#### 4) No Calculator

The continuous function  $f$  is defined on the interval  $-4 \leq x \leq 3$ .

The graph of  $f$  consists of two quarter circles and one line segment, as shown in the figure above.

$$\text{Let } g(x) = 2x + \int_0^x f(t) dt.$$

- (a) Find  $g(-3)$ . Find  $g'(x)$  and evaluate  $g'(-3)$ .
- (b) Determine the  $x$ -coordinate of the point at which  $g$  has an absolute maximum on the interval  $-4 \leq x \leq 3$ .  
Justify your answer.
- (c) Find all values of  $x$  on the interval  $-4 < x < 3$  for which the graph of  $g$  has a point of inflection. Give a reason for your answer.
- (d) Find the average rate of change of  $f$  on the interval  $-4 \leq x \leq 3$ . There is no point  $c$ ,  $-4 < c < 3$ , for which  $f'(c)$  is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.



Graph of  $f$