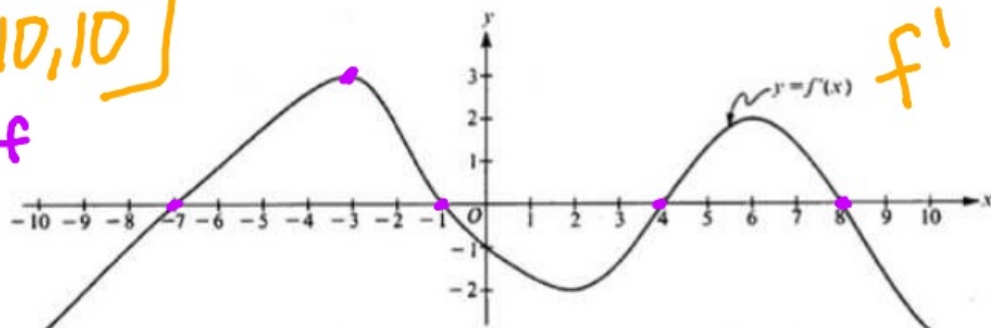


## Interpreting Derivative Graphs

$[-10, 10]$

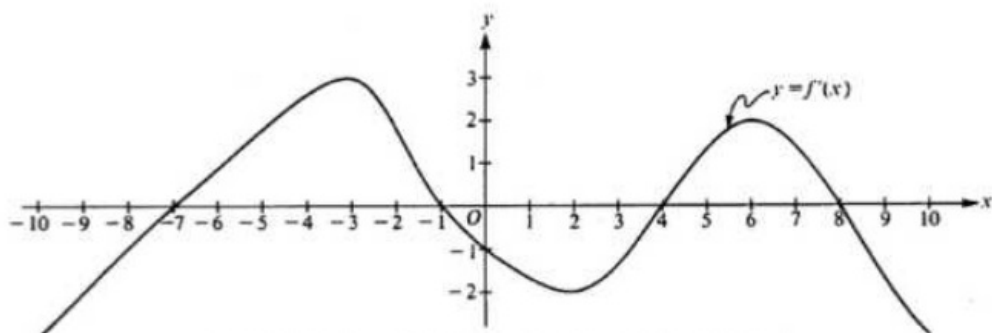
slope of  
 $f$  at  
 $x = -3$   
is 3



Note: This is the graph of the derivative of  $f$ , not the graph of  $f$ .

Is  $f(x)$  differentiable on the entire interval?

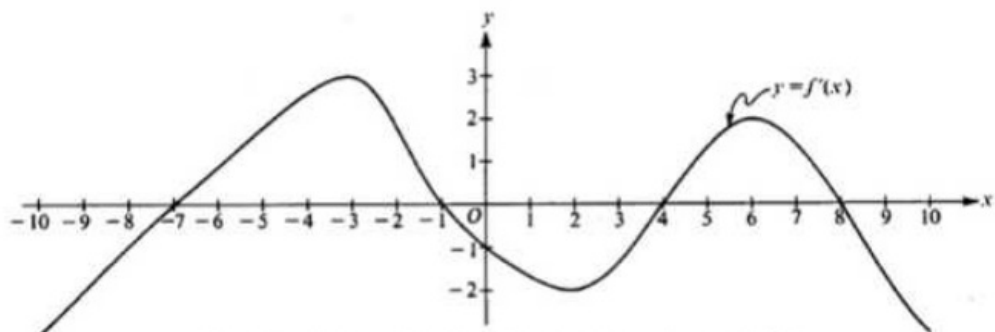
Yes,  $f'(x)$  is continuous



Note: This is the graph of the derivative of  $f$ , not the graph of  $f$ .

Is  $f(x)$  continuous on the entire interval?

Yes ; diff. implies continuity

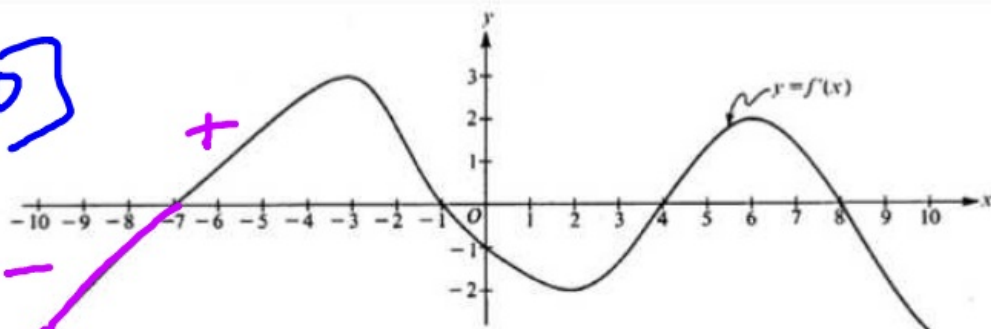


Note: This is the graph of the derivative of  $f$ , not the graph of  $f$ .

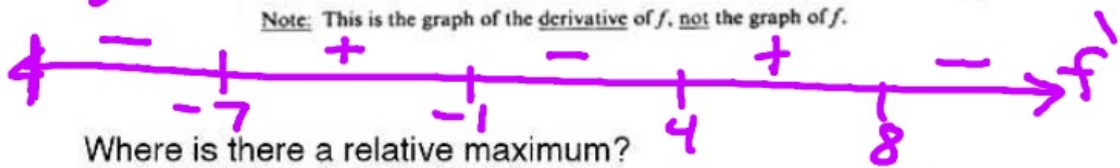
Where is the derivative ~~zero~~ *of  $f$  zero?*

$$x = -7, -1, 4, 8 \quad (f' = 0)$$

$[-10, 10]$



Note: This is the graph of the derivative of  $f$ , not the graph of  $f$ .

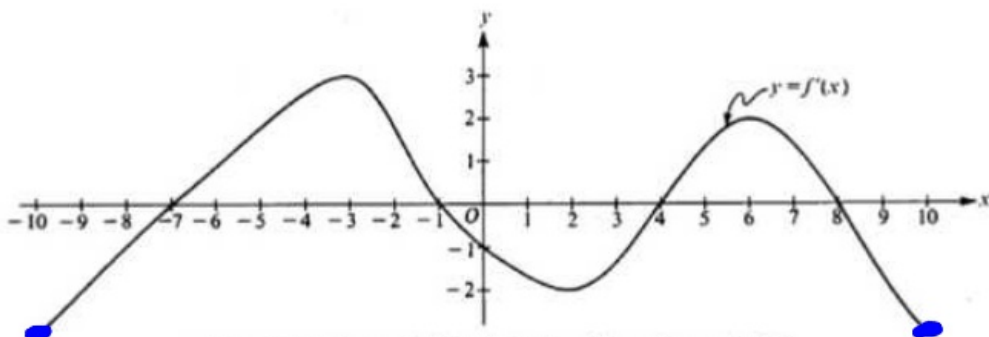


Where is there a relative maximum?

rel. max @  $x = -1, 8$   $f'$  changes from pos. to neg.

rel. min @  $x = -7, 4$   $f'$  changes from neg. to pos.

$f$  incr / decr.

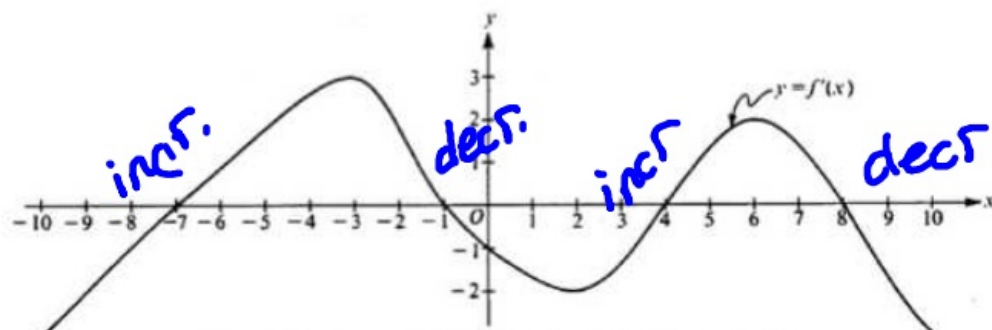


Note: This is the graph of the derivative of  $f$ , not the graph of  $f$ .

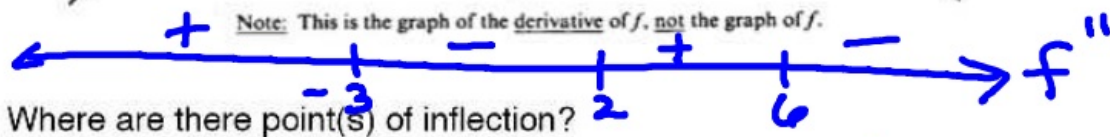
Where is  $f(x)$  increasing?  $(-7, -1) \cup (4, 8)$

because  $f' > 0$

decr.  $(-10, -7) \cup (-1, 4) \cup (8, 10)$   $f' < 0$



Note: This is the graph of the derivative of  $f$ , not the graph of  $f$ .

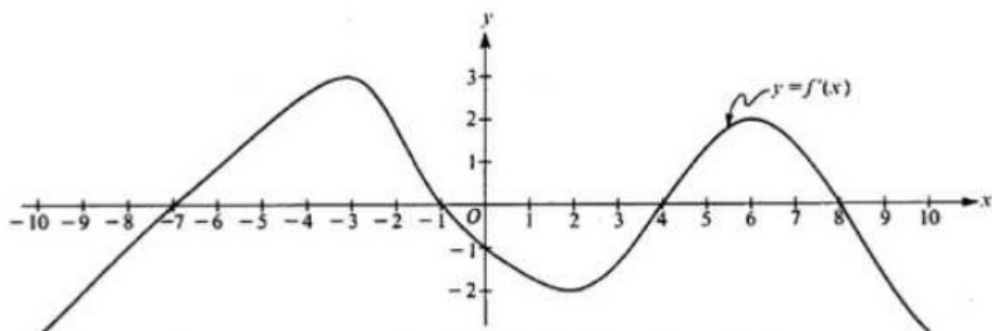


Where are there point(s) of inflection?

$x = -3, 2, 6$   
 $f'$  changes from incr. to decr.  
 or decr. to incr.

slope of  $f'$  changes  
 signs

$f'$  has relative extrema  
 at these points



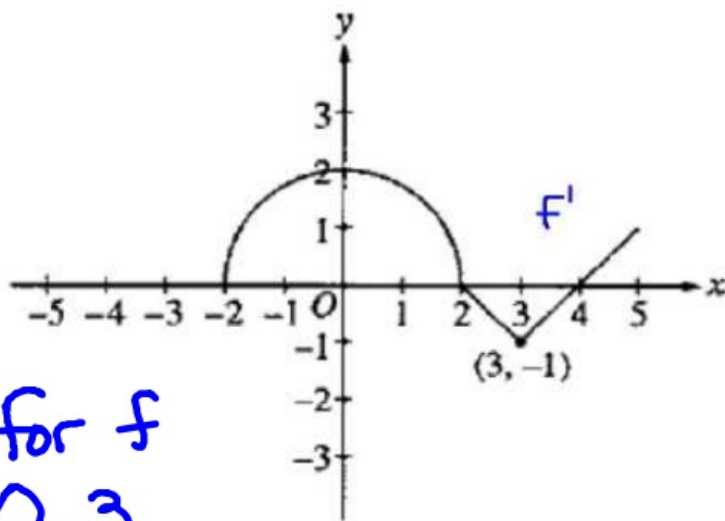
Note: This is the graph of the derivative of  $f$ , not the graph of  $f$ .

Where is  $f(x)$  concave up?  $(-10, -3) \cup (2, 6)$   
 slope of  $f' > 0$

CCD:  $(-3, 2) \cup (6, 10)$  slope of  $f' < 0$



$[-2, 5]$



POI for  $f$

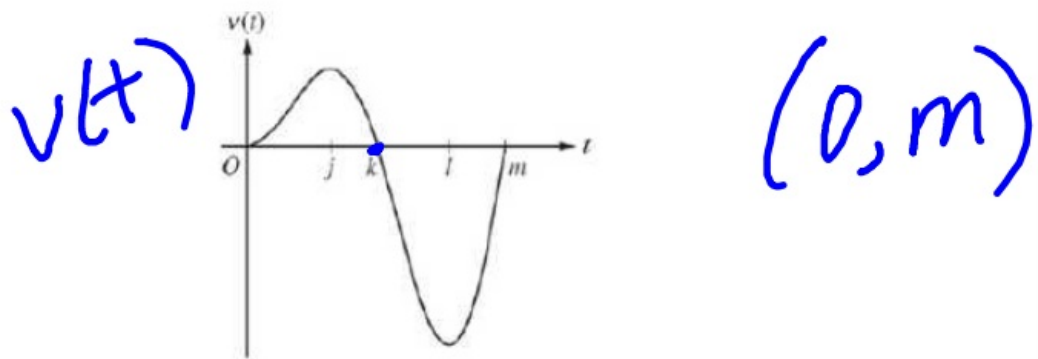
$x = 0, 3$

$f'$  has rel. extrema at  
these points

### I. Review: Motion Along A Line

If  $x(t)$  represents the position of a particle along the  $x$ -axis at any time  $t$ , then the following statements are true.

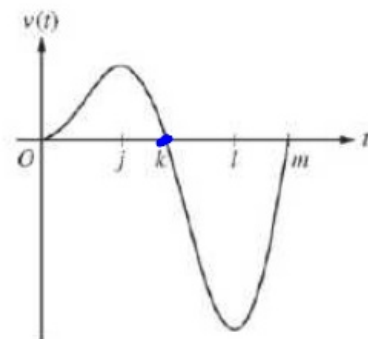
1. "Initially" means  $t = 0$ .
2. "At rest" means  $x'(t) = 0$ .
3. If the velocity of the particle is positive, then the particle is moving to the right.
4. If the velocity of the particle is neg., then the particle is moving to the left.
5. To find average velocity over a time interval, divide the change in position by the change in time.
6. Instantaneous velocity is the velocity at a single moment or point in time.
7. If the acceleration of the particle is positive, then velocity is increasing.  $x'$
8. If the acceleration of the particle is neg., then the velocity is decreasing.
9. In order for a particle to change direction, the velocity must change signs.
10. Speed is the abs. value of velocity.



1) State the value(s) of  $t$  where the particle is at rest.

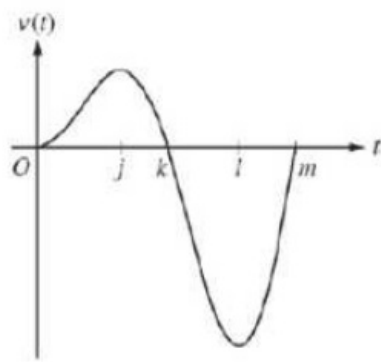
$$t = k$$

$v(t)$



2) State the value(s) of  $t$  where the particle is changing direction.

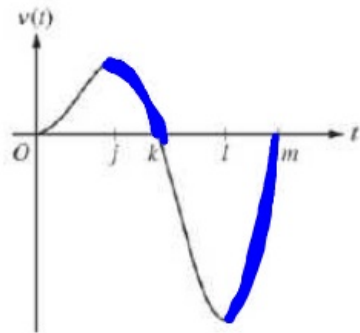
$t = k$   $v(t)$  changes signs



3) State the interval(s) where the particle is moving to the right.

$(0, k); v(t) > 0$

left :  $(k, m); v(t) < 0$

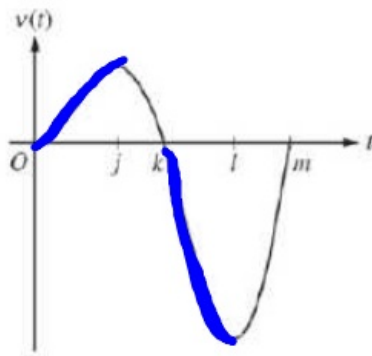


$v(t)$   
 $v'(t)$   
 opposite  
 signs

5) State the interval(s) where the particle is slowing down.

           (speed decreasing)  
 $(j, k)$ ;  $v(t) > 0$  slope of  $v(t) < 0$   
 $(l, m)$ ;  $v(t) < 0$  slope of  $v(t) > 0$

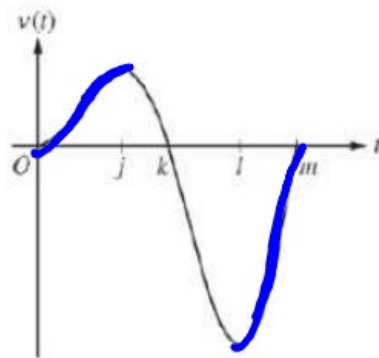
Speed



$v(t)$   
and  
 $v'(t)$   
same sign

6) State the interval(s) where the particle is speeding up.

$(0, j)$ ;  $v(t) > 0$  slope of  $v(t) > 0$   
 $(k, e)$ ;  $v(t) < 0$  slope of  $v(t) < 0$



7) State the interval(s) where the velocity is increasing.

$(0, j)$   
 $(l, m)$

slope of  $v(t) > 0$



2. Analytical (NO CALCULATOR)

A particle moves along the x-axis so that at any time  $t$  its position is given by:

$$x(t) = t^3 - 6t^2 + 9t + 11$$

b) At  $t=1$ , is the velocity of the particle increasing or decreasing? JYA.

$$x''(1) < 0$$

$\therefore$  vel. is decreasing  
at  $t=1$

2. Analytical (NO CALCULATOR)

A particle moves along the x-axis so that at any time  $t$  its position is given by:

$$x(t) = t^3 - 6t^2 + 9t + 11$$

$$x'(5) > 0$$
$$x''(5) > 0$$

e) At  $t=5$ , is the speed of the particle increasing or decreasing? JYA.

$x'(5)$  and  $x''(5)$  have the same  
Sign (speeding up)

↗