# Extra Midterm Review (Chapter 1 and 2)

1. The length of a rectangle is decreasing at the rate of 2 cm/sec while the width is increasing at the rate of 2 cm/sec. When the length is 12 cm and the width is 5 cm, find the rates of change of:
   a) the area
   b) the perimeter
   c) the length of a diagonal of the rectangle

2. A 13 ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at a rate of 5 ft/sec.
   a) How fast is the top of the ladder sliding down the wall at that moment?
   b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing at that moment?
   c) At what rate is the angle between the ladder and the ground changing at that moment?

3.

Find the value of a if f(x) is a continuous function.

$$f(x) = \begin{cases} 3x + a, & x \le -3 \\ ax^2 + 4, & x > -3 \end{cases}$$

Evaluate.

4.

$$\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{2} + h\right) - 1}{h}$$

5.

$$\lim_{x \to e} \frac{\ln x - 1}{x - e}$$

6.

Let $f$ be the function defined as follows:

$$f(x) = \begin{cases} |x-1| + 2, & \text{for } x < 1 \\ ax^2 + bx, & \text{for } x \ge 1, \text{ where } a \text{ and } b \text{ are constants.} \end{cases}$$

(a) If $a = 2$ and $b = 3$, is $f$ continuous for all $x$? Justify your answer.

(b) Describe all values of $a$ and $b$ for which $f$ is a continuous function.

(c) For what values of $a$ and $b$ is $f$ both continuous and differentiable?

7.

If $3x^2 - 4xy = 1$, then when $x = 1$, $\frac{dy}{dx} =$

8.

Let $f$ and $g$ be functions that are differentiable everywhere. If $g$ is the inverse function of $f$ and if $g(-2) = 5$ and $f'(5) = -\frac{1}{2}$, then $g'(-2) =$

Differentiate.

| | | | |
|---|---|---|---|
| 9. $f(x) = \sqrt{x^2 + 8}$ | 10. $f(x) = \ln(xe^{7x})$ | 11. $f(x) = \cot(6x)$ | 12. $f(x) = \cos^4 x - 2x^2$ |
| 13. $f(x) = \frac{(3x^2 - \pi x)^4}{6}$ | 14. $f(x) = \frac{6}{(3x^2 - \pi)^4}$ | 15. $f(x) = (e^{2x} + e)^{\frac{1}{2}}$ | 16. $\left[\arctan(2x)\right]^{10}$ |
| 17. $f(x) = \arcsin(2^x)$ | 18. $f(x) = \frac{2x^4 + 3x^2 - 1}{x^2}$ | 19. $f(x) = \left[\ln(5x^2 + 9)\right]^3$ | 20. $f(x) = \sec^2 x \cdot \tan x$ |

Answers

1.
a. dA/dt = 14cm^2/sec
b. dP/dt = 0 cm/sec
c. dD/dt = -14/13 cm/sec

2
a. -12 ft/sec
b. -119/2 ft^2/s
c. -1 radian/sec

3. a = -13/8

1.
a. dA/dt = 14cm^2/sec
b. dP/dt = 0 cm/sec
c. dD/dt = -14/13 cm/sec

2
a. -12 ft/sec
b. -119/2 ft^2/s
c. -1 radian/sec

4. 0

5. 1/e

6.

(a) No, $f$ is not continuous for all $x$ since it is not continuous at $x = 1$.

$\lim_{x\to 1^-} f(x) = 2,\ f(1) = 5$

$\therefore \lim_{x\to 1^-} f(x) \neq f(1)$

or

$\lim_{x\to 1^-} f(x) = 2,\ \lim_{x\to 1^+} f(x) = 5$

$\therefore \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

(b) $f(1) = a + b$

or

$\lim_{x\to 1^+} f(x) = a + b$

The function $f$ is continuous when $a + b = 2$.

(c) $f'(x) = \begin{cases} -1 & \text{if } x < 1 \\ 2ax + b & \text{if } x > 1 \end{cases}$

To be continuous and differentiable at $x = 1$, must have

$a + b = 2$

$2a + b = -1$

Therefore $a = -3$ and $b = 5$.

7. 1

8. -2