Solids With Known Cross Sections

1. (Calculator)Find the volume of the solid whose base is bounded by the graphs of $y = 8 - x^2$ and $y = x^2$, with the indicated cross sections perpendicular to the x-axis:
   a. Squares
   b. Semi-Circles
   c. Equilateral Triangles

2. Set up the integral, (DO NOT SOLVE!), to find the volume of each solid described. The base of the volume is the region bounded by the curve $y = 2 + \sin x$, the y-axis, x-axis and $x = \frac{3\pi}{2}$.
   The cross sections perpendicular to the x-axis:
   a. Rectangles with height 1
   b. Quarter-Circles
   c. Isosceles Right Triangles (hypotenuse on base)

3. (Calculator) The base of a solid is the region bounded by $y = x^2 - 2$ and $y = 3x + 8$. Find the volume of each solid whose cross sections perpendicular to the

   a. x-axis are semicircles
   b. x-axis are rectangles with height of 4

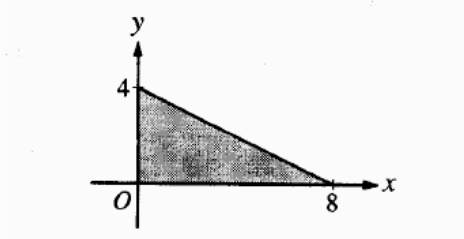
4. (Calculator) The base of a solid is the region bounded by $y = \frac{1}{2}x^3$, y= 0, and x = 2. Find the volume of each solid whose cross sections perpendicular to the

   a. x-axis are semi-circles
   b. y-axis are equilateral triangles

5.
The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

(A) $\frac{\left(1 - e^{-6}\right)}{2}$ (B) $\frac{1}{2}e^{-6}$ (C) e^{-6} (D) e^{-3} (E) $1 - e^{-3}$

6. Calculator



The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

7. Calculator

The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3 - 1)$

8.

The base of a solid is the region in the first quadrant enclosed by the parabola $y = 4x^2$, the line $x = 1$, and the x-axis. Each plane section of the solid perpendicular to the x-axis is a square. The volume of the solid is

(A) $\frac{4\pi}{3}$ (B) $\frac{16\pi}{5}$ (C) $\frac{4}{3}$ (D) $\frac{16}{5}$ (E) $\frac{64}{5}$

9.

The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

(A) $\pi\int_0^2 (2-y)^2\,dy$

(B) $\int_0^2 (2-y)\,dy$

(C) $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2\,dx$

(D) $\int_0^{\sqrt{2}} \left(2-x^2\right)^2\,dx$

(E) $\int_0^{\sqrt{2}} \left(2-x^2\right)dx$

ANSWERS

1.

a) 136.533
b) 53.617
c) 59.121

2.

a) $\int_0^{3\pi/2} (2+\sin x)dx$

b) $\int_0^{3\pi/2} \frac{(2+\sin x)^2}{4} dx$

c) $\int_0^{3\pi/2} \frac{(2+\sin x)^2\sqrt{3}}{4} dx$

3.

a) 220.003
b) 228.667

4.

a) 1.795
b) 0.693

5. A
6. C
7. C
8. D
9. B