AB Calculus Ch 2a Review

Part I: Free Response

Find the derivative by the limit process.

1) $h(x) = 6x - x^2$	2) $m(x) = \sqrt{x-4}$

Find f'(2) using the alternate form of the derivative.

3) $f(x) = 4x^2 - 6x + 3$

Sketch each function. State the interval(s) where each function is differentiable. If the function is not differentiable at a point, explain using limits.

4) $f(x) = \lvert x-3 \rvert + 2$	5) $g(x) = (x+2)^{2/3}$	6) $h(x) = -\sqrt[3]{x} - 2$

Differentiate. Re-write without using negative exponents.

7) $y = \tan(3x)$	14) $h(t) = \sin^3 4t$
8) $f(t) = \dfrac{5t^3 + 7t^2 - 4}{t}$	15) $y = (\ln x)^4$
9) $f(x) = x^2 2^{-x}$	16) $f(x) = x\cos(x^2)$
10) $y = 4\csc x - 2\cot x + \pi^3$	17) $y = -\dfrac{2}{\sqrt{4x+7}}$
11) $g(x) = \dfrac{1}{x^2} - \dfrac{2}{\sqrt{x}}$	18) $s(t) = \dfrac{\ln t^2}{t}$
12) $y = xe^x - e^x$	19) $y = \sqrt{1 - e^{4x}}$
13) $y = \dfrac{x-4}{x^2+1}$	20) $g(x) = 2e^{1-x^2}$

Find the slope at the given value of c.

21) $f(x) = \sqrt[3]{1-x}$; c = -7	22) $g(x) = \sec(5x)$; $c = \dfrac{\pi}{3}$

Write the equation of the tangent line at the given point.

23) $f(x) = \dfrac{\tan x}{x}$; $(\pi, 0)$	24) $y = \log_3 x$; (3, 1)

Determine the point(s) where the function has horizontal tangent(s).

25) $y = x^2 e^{-x}$

Find $\frac{d^2y}{dx^2}$. Factor and simplify completely.

26) $y = \ln|\sin x|$

27) $f(x) = \csc x$

28) $g(x) = (x^2+1)^5$

Given the following information, find the derivatives indicated.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	5	-1	3	2
3	6	2	-2	-4
5	-3	4	5	-1

29) $h(x) = f(x)g(x)$ *Find h'(3)*

30) $m(x) = f(g(x))$; *Find m'(2)*

31) $n(x) = \frac{g(x)}{f(x)}$ Find n'(2)

32) $p(x) = (g(x))^3$; Find p'(5)

Find k such that the line y = -6x + 1 is tangent to the graph of the function.

33) $f(x) = k - x^2$

Is f(x) continuous and differentiable? Explain.

34) $f(x) = \begin{cases} 2-x & x \le 1 \\ x^2 - 2x + 2 & x > 1 \end{cases}$

Find a and b so that g(x) is continuous and differentiable.

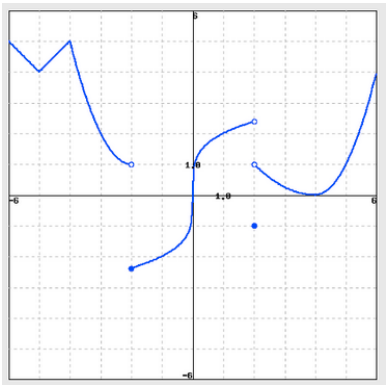
35) $g(x) = \begin{cases} x^2 & x \le 2 \\ ax + b & x > 2 \end{cases}$

Evaluate.

36) $\lim_{h \to 0} \frac{3(x+h)^2 - 4(x+h) - 3x^2 + 4x}{h}$

37) $\lim_{h \to 0} \frac{\tan\left(h + \frac{\pi}{4}\right) - 1}{h}$

38) Use the given graph to determine the x-values in the open interval (-6, 6) which are not differentiable.



Part II: Multiple choice

1)

Which of the following functions are not differentiable at $x = \frac{2}{3}$?

I. $f(x) = \sqrt[3]{x-2}$ II. $g(x) = |3x-2|$ III. $h(x) = |9x^2 - 4|$

A) I only

B) II only

C) I and II only

D) II and III only

E) I and III only

2) $\frac{d}{dx}\sqrt{\sin 2\theta}$

a. $\frac{1}{\sqrt{\sin 2\theta}}$ b. $\frac{\cos 2\theta}{\sqrt{\sin 2\theta}}$ c. $\frac{2\cos 2\theta}{\sqrt{\sin 2\theta}}$ d. $2\cos 2\theta\sqrt{\sin 2\theta}$ e. $\cos 2\theta\sqrt{\sin 2\theta}$

3)

$$\lim_{x\to 0}\frac{\left(3e^x - x\right)-3}{x}$$

The above limit represents $f'(c)$, the derivative of some function $f(x)$ at some $x = c$. What are $f(x)$ and $x = c$?

(A) $f(x) = e^x - x$, $c = 3$ (B) $f(x) = 3e^x$, $c = 0$ (C) $f(x) = 3e^x - x - 3$, $c = 0$

(D) $f(x) = 3e^x - x$, $c = 0$ (E) $f(x) = 3e^x - x$, $c = 3$

4)

If $f(x) = \frac{x+1}{x-1}$, which of the following statements is true?

I. $\lim_{x\to 1} f(x) = -\infty$ II. $\lim_{x\to 1} f'(x) = -\infty$ III. $\lim_{x\to 1} f''(x) = -\infty$

S) I only T) II only V) III only W) I and II only X) II and III only

5)

The 30th derivative of $\cos 2x$ is

A. $60\sin 2x$ B. $-60\sin 2x$ C. $-60\cos 2x$ D. $-2^{30}\cos 2x$ E. $2^{30}\cos 2x$

6)

$$\lim_{h\to 0}\frac{\ln(e+h)-1}{h} \text{ is}$$

(A) $f'(e)$, where $f(x) = \ln x$ (B) $f'(e)$, where $f(x) = \frac{\ln x}{x}$ (C) $f'(1)$, where $f(x) = \ln x$

(D) $f'(1)$, where $f(x) = \ln(x+e)$ (E) $f'(0)$, where $f(x) = \ln x$

7)

If $f(x) = -x^2 + x$, then which of the following expressions represents $f'(x)$?

A) $\lim_{h \to 0} \frac{(-x^2 + x + h) - (-x^2 + x)}{h}$

B) $\lim_{h \to x} \frac{(-x^2 + x + h) - (-x^2 + x)}{h}$

C) $\frac{[-(x+h)^2 + (x+h)] - (-x^2 + x)}{h}$

D) $\lim_{h \to 0} \frac{[-(x+h)^2 + (x+h)] - (-x^2 + x)}{h}$

E) None of these

8)

$\frac{d}{dx}\left(\ln e^{2x}\right) =$

(A) $\frac{1}{e^{2x}}$ (B) $\frac{2}{e^{2x}}$ (C) $2x$ (D) 1 (E) 2

If $f(3) = 2$, $g(3) = -\frac{3}{2}$, $f'(3) = -2$, $g'(3) = 5$, and $h(x) = \left[f(x) + 2g(x)\right]^3$, find $h'(3)$.

(A) -24 (B) 24 (C) 1 (D) -1 (E) 42

10)

Which of the following is the equation of the normal line to the function $f(x) = x^2 + 3x - 5$ at $x = 1$?

(A) $5x - y = -4$ (B) $x - 5y = -4$ (C) $5x + y = -4$ (D) $x + 5y = -4$ (E) $-5x + y = -4$

11)

. If $y = e^{-x} \ln x$ then $\frac{dy}{dx}$ when $x = 1$ is

A) 0 B) Does not exist C) $\frac{2}{e}$ D) $\frac{1}{e}$ E) e

12)

If $y = \ln(x\sqrt{x^2 + 1})$, then $\frac{dy}{dx} =$

A) $1 + \frac{x}{x^2 + 1}$ B) $1 + \frac{1}{x\sqrt{x^2 + 1}}$ C) $\frac{2x^2 + 1}{x\sqrt{x^2 + 1}}$ D) $\frac{2x^2 + 1}{x(x^2 + 1)}$ E) $\frac{x^2 + x + 1}{x(x^2 + 1)}$

Part I Answers

For #1 and #2, use $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

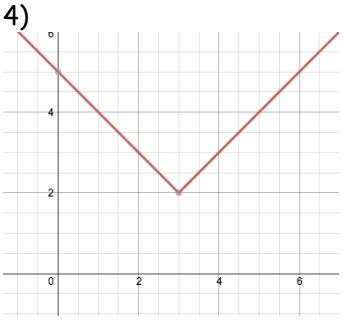
1) $h'(x)=6-2x$

2) $m'(x)=\frac{1}{2\sqrt{x-4}}$

For #3, use $f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$

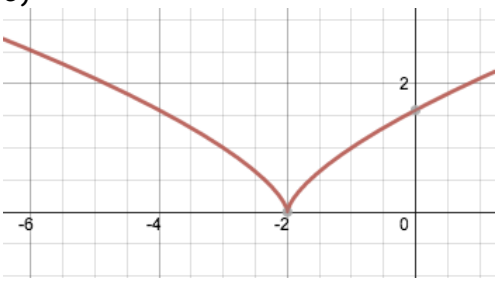
3) $f'(2)=10$

4)



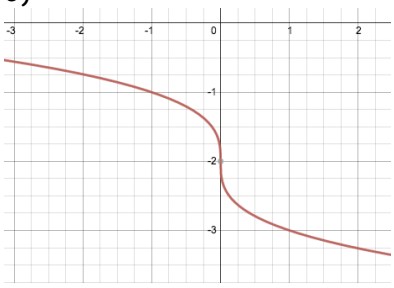
$(-\infty,3)\cup(3,\infty)$

5)



$(-\infty,-2)\cup(-2,\infty)$

6)



$(-\infty,0)\cup(0,\infty)$

7) $y'=3\sec^2(3x)$

8) $f'(t)=10t+7+\frac{4}{t^2}$

9) $f'(x)=x2^{-x}(2-x\ln 2)$

10) $y'=-4\csc x\cot x+2\csc^2 x$

11) $g'(x)=\frac{-2}{x^3}+\frac{1}{x^{3/2}}$

12) $y'=xe^x$

13) $y'=\frac{-x^2+8x+1}{(x^2+1)^2}$

14) $h'(t)=12\sin^2 4t\cos 4t$

15) $y'=\frac{4(\ln x)^3}{x}$

16) $f'(x)=\cos(x^2)-2x^2\sin(x^2)$

17) $y'=\frac{4}{(4x+7)^{3/2}}$

18) $s'(t)=\frac{2(1-\ln t)}{t^2}$

19) $y'=\frac{-2e^{4x}}{\sqrt{1-e^{4x}}}$

20) $g'(x)=-4xe^{1-x^2}$

21) $f'(-7) = -\frac{1}{12}$ 22) $g'\left(\frac{\pi}{3}\right) = -10\sqrt{3}$ 23) $y = \frac{1}{\pi}(x - \pi)$ 24) $y - 1 = \frac{1}{3\ln 3}(x-3)$ or $y - 1 = \frac{1}{\ln 27}(x-3)$ 25) (0, 0) and $\left(2, \frac{4}{e^2}\right)$ 26) $y'' = -\csc^2 x$ 27) $f''(x) = \csc x(\cot^2 x + \csc^2 x)$ 28) $g''(x) = 10(x^2+1)^3(9x^2+1)$	29) -28 30) 4 31) 13/25 32) -75 33) k = -8 34) Continuous but not differentiable. 35) a = 4 b = -4 36) 6x - 4 37) 2 38) -5, -4, -2, 0, 2

Part II Answers

1) D 2) B 3) D 4) T 5) D 6) A	7) D 8) E 9) B 10) D 11) D 12) D