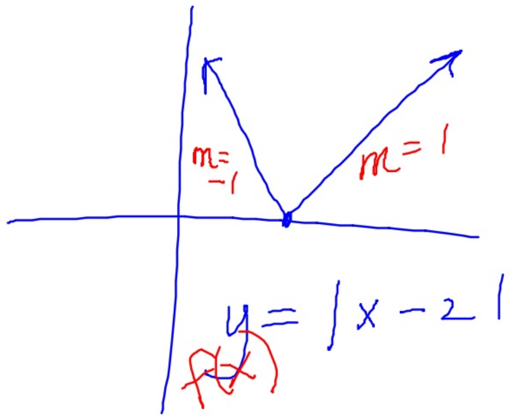
Chapter 1 Topic to review: IVT

The test is ALL of chapter 2 so review

Definition of derivative
Alternate Form of Derivative
Recognizing a limit as a difference quotient (derivative)
Differentiability
ALL derivative formulas
Tangent line/normal line
Motion on a line (include units of measure)
Average velocity (average rate of change) No calculus needed!

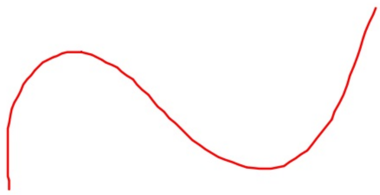
PLUS new topics: *implicit differentiation*
log differentiation
Derivative of inverse
Inverse trig derivatives
related rates

Need extra practice? Use the previous ch 2a review and textbook qustions



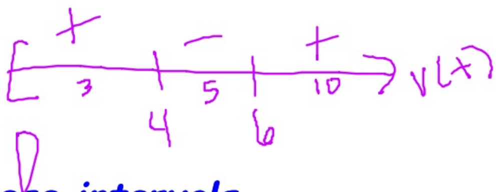
$$\lim_{x \to 2^-} f'(x) \neq \lim_{x \to 2^+} f'(x)$$

$$-1 \neq 1$$



$$v(t) = (t-4)(t-6) \qquad t \geq 0$$

When is the velocity positive? Justify.



(0,4) and (6, inf) because v(t) > 0 on these intervals.

When does the particle change direction?

At t = 4 and 6 because the sign of v(t) changes at these times.

$g(x) = x^3 + 4x + 1$

$(g^{-1})'(-4) = \frac{1}{7}$

$g'(x) = 3x^2 + 4$

$g: (-1, -4)$

$g'(-1) = 7$

$g^{-1}: (-4, -1)$

Write an equation of the normal line to $g^{-1}(x)$ at $x = -4$.

$y + 1 = -7(x + 4)$

numerical

$$x^2 + y^2 = 9 \Longrightarrow y = \pm\sqrt{9 - x^2}$$

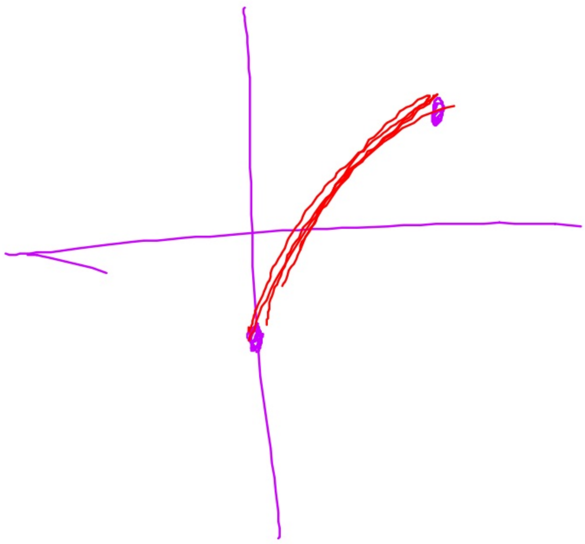
$$\frac{dy}{dx} = \frac{-x}{y} \qquad y' = \pm\frac{1}{2}(9 - x^2)^{-1/2}(-2x)$$

$$= \frac{\pm(-x)}{\sqrt{9 - x^2}}$$

y

$$-\frac{x}{y}$$

Since $f(x)$ is continuous on $[a,b]$ and $f(a) < 0 < f(b)$ by IVT there must exist a value c on $[a,b]$ such that $f(c) = 0$



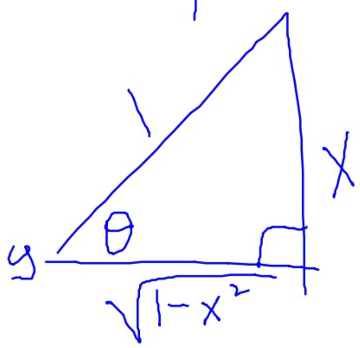
$y = \arcsin u$ $\qquad$ $y' = \dfrac{u'}{\sqrt{1-u^2}}$

$y = \arctan u$ $\qquad$ $y' = \dfrac{u'}{1+u^2}$

$y = \operatorname{arcsec} u$ $\qquad$ $y' = \dfrac{u'}{|u|\sqrt{u^2-1}}$

$$\sin(y) = \sin(\arcsin x)$$

$$y = \arcsin \frac{x}{1}$$



$$\left(\sin y = x\right)\frac{d}{dx}$$

$$\cos y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\frac{\sqrt{1-x^2}}{1}} = \frac{1}{\sqrt{1-x^2}}$$

$$\lim_{x \to \pi} \frac{\cos x + 1}{x - \pi}$$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$f(x) = \cos x$

$c = \pi$

$f'(x) = -\sin x$

$f'(\pi) = 0$

20c.)

$$\frac{dV}{dt} = \frac{dr}{dt}$$

$$4\pi r^2 \frac{dr}{\cancel{dt}} = \frac{dr}{\cancel{dt}}$$

$$4\pi r^2 = 1$$

$$r^2 = \frac{1}{4\pi}$$

$$r = \frac{1}{2\sqrt{\pi}}$$

b.)

$$V = 36\pi$$

$$\frac{4}{3}\pi r^3 = 36\pi$$

$$r = 3$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

3) $\frac{dr}{dt} = \frac{dh}{dt} = \frac{1}{2}$

$$\frac{d}{dt}\left(V = \frac{1}{3}\pi r^2 h\right)$$

$$\frac{dV}{dt} = \frac{\pi}{3}\left(r^2 \frac{dh}{dt} + h \cdot 2r \frac{dr}{dt}\right)$$

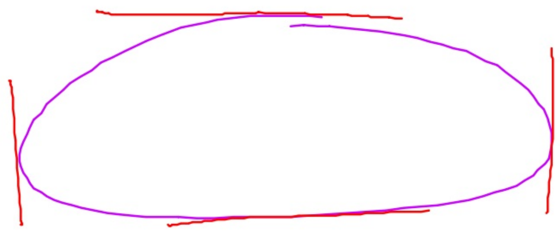
$$\ln(y) = \ln\left(\frac{\sqrt{x}}{(x+1)^2}\right)$$

$$\left(\ln y = \frac{1}{2}\ln x - 2\ln(x+1)\right)\frac{d}{dx}$$

$$\frac{1}{\cancel{y}}\cdot\frac{dy}{dx} = \left(\frac{1}{2x} - \frac{2}{x+1}\right)y$$

$$\frac{dy}{dx} = \left(\frac{1}{2x} - \frac{2}{x+1}\right)\left(\frac{\sqrt{x}}{(x+1)^2}\right)$$

$$\frac{x}{y}$$



23b.

$A = \text{square} - \text{circle}$

$A = 4r^2 - \pi r^2$