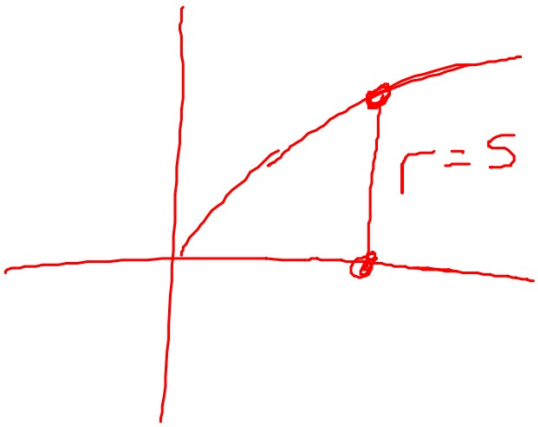
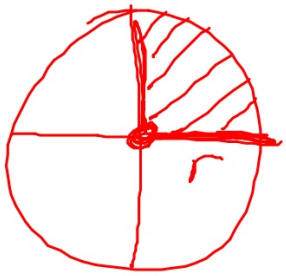
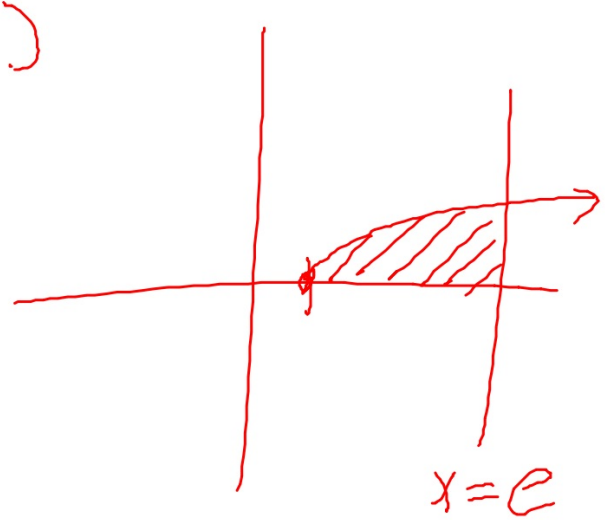


$$\frac{1}{4}\pi r^2$$

$$\frac{1}{4}\pi s^2$$



5)



$\perp$ x-axis

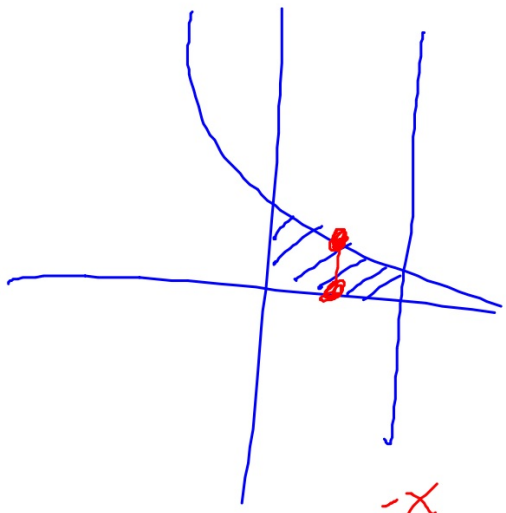
squares

$$S = \left(\sqrt{\ln x} - 0\right)$$

$$\int_1^e s^2\,dx = \int_1^e \left(\sqrt{\ln x}\right)^2 dx$$

C.

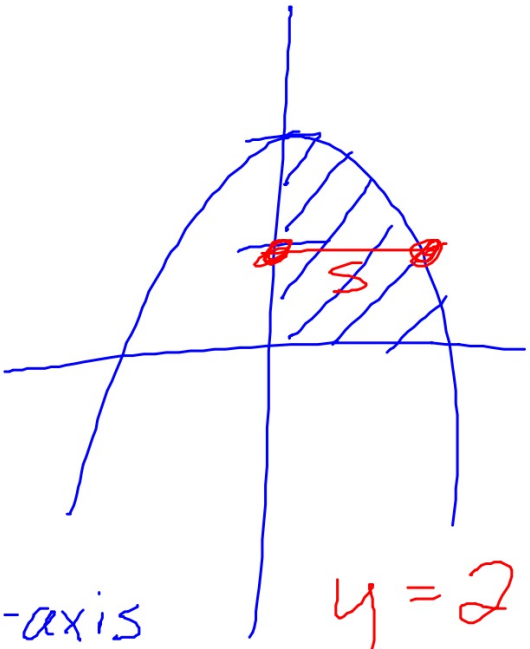
3) $y = e^{-x}$, $x = 3$, x-axis, y-axis



$$\int_0^3 e^{-2x}\,dx$$

$$-\frac{1}{2}e^{-2x}\Big|_0^3$$

$$-\frac{1}{2}\left(e^{-6} - 1\right)$$



$\int_0^2 (S^2)\,dy$

$\int_0^2 (2-y)\,dy$

$\perp$ y-axis

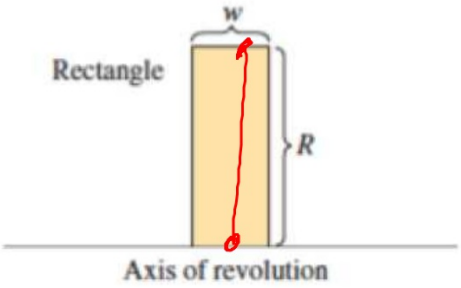
$y = 2 - x^2$

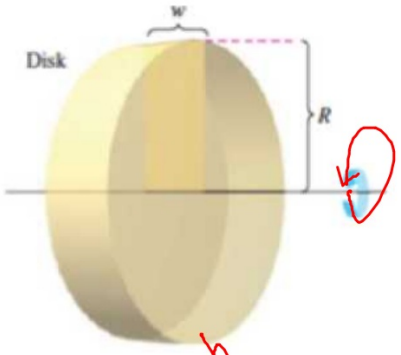
$\pm\sqrt{2-y} = X$

$S = \sqrt{2-y}$

6.2 Volume: Solids Of Revolution

Disk Method





$$A_{\text{circle}} = \pi r^2$$

$$V_{\text{cylinder}} = \pi r^2 h$$

$\pi \int_a^b r^2 dx$ or $\pi \int_a^b r^2 dy$

Disk Method

To find the volume of a solid of revolution with the **disk method,** use one of the formulas below. (See Figure 6.15.)

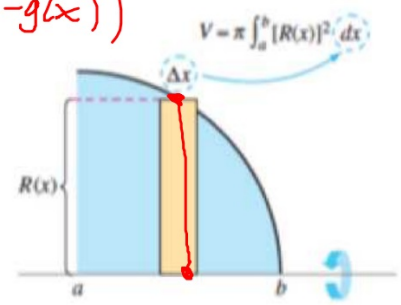
Horizontal Axis of Revolution	*Vertical Axis of Revolution*
$\text{Volume} = V = \pi \int_a^b [R(x)]^2\, dx$	$\text{Volume} = V = \pi \int_c^d [R(y)]^2\, dy$

$R(x) = (f(x) - g(x))$

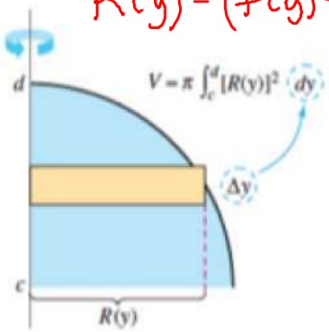
$R(y) = (f(y) - g(y))$

$(x-5)^2$

$(5-x)^2$

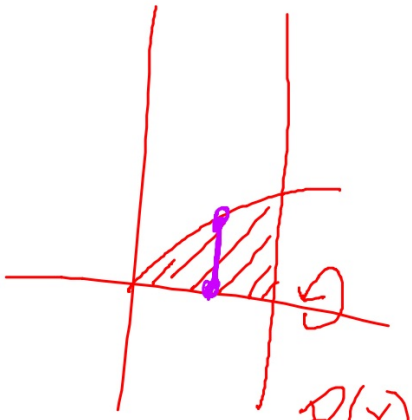


Horizontal axis of revolution



Vertical axis of revolution

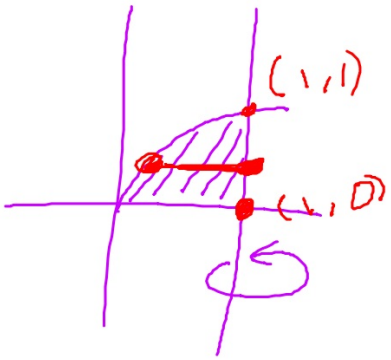
ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$, $x = 0$, $x = 1$ and the x-axis about the x-axis.



$$V = \pi \int_0^1 (\sqrt{x})^2 \, dx = \pi \int_0^1 x \, dx$$

$R(x) = (\sqrt{x} - 0)$

ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x},\ x = 0,\ x = 1$ and the x-axis about the line x=1.



$$V = \pi \int_0^1 \left(1 - y^2\right)^2 dy$$

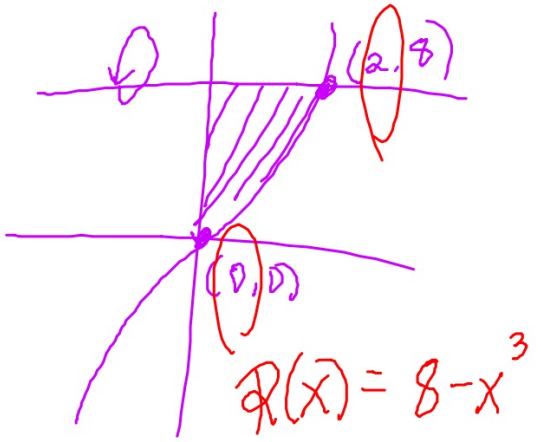
$R(y) = \left(1 - y^2\right)$

$y^2 = \sqrt{x}^2$

$y^2 = x$

ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded in the first quadrant by the graphs of $y = x^3$ and $y = 8$ about the line y=8.

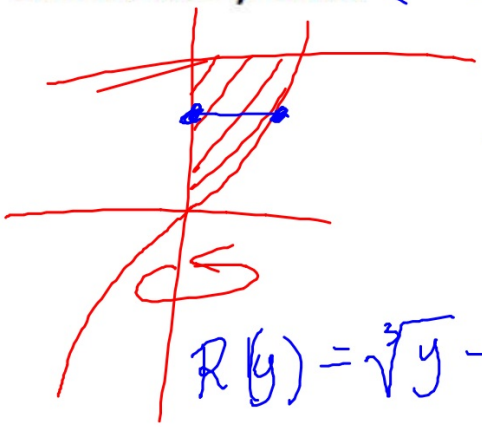
horiz. line



$$\pi \int_0^2 (8 - x^3)^2 \, dx$$

ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded in the first quadrant by the graphs of $y = x^3$ and $y = 8$ about the y-axis.

vertical line



$R(y) = \sqrt[3]{y} - 0$

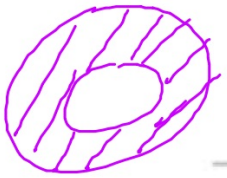
$$\pi \int_0^8 \left(\sqrt[3]{y}\right)^2 dy$$

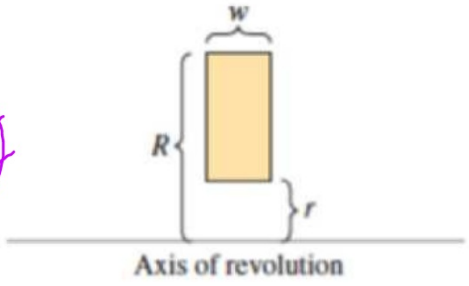
ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded in the first quadrant by the graphs of $x = y - y^2$ and the y-axis about the y-axis.

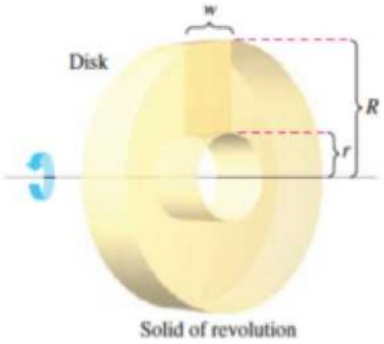
ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded by the graphs of $f(x)=2-x^2$ and $g(x)=1$ about the line y=1.

ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded by the graphs of $f(x)=2-x^2$, $g(x)=1$ and $h(x)=0$ about the x-axis.

Washer Method







$$A_{\text{washer}} = \pi R^2 - \pi r^2 = \pi\left(R^2 - r^2\right)$$

$$V_{\text{cylinder}} = \pi R^2 h - \pi r^2 h = \pi\left(R^2 - r^2\right)h$$

Washer Method

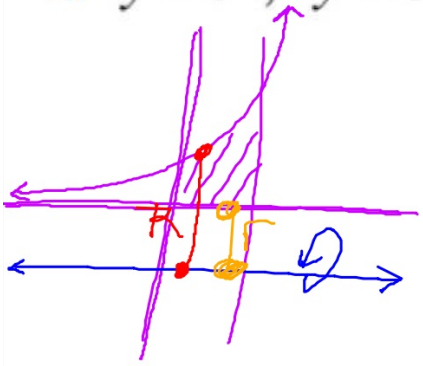
Horizontal Axis of Revolution

$$V = \pi \int_a^b \left(\left[R(x) \right]^2 - \left[r(x) \right]^2 \right) dx$$

Vertical Axis of Revolution

$$V = \pi \int_a^b \left(\left[R(y) \right]^2 - \left[r(y) \right]^2 \right) dy$$

ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded by the graphs of $y = e^x$, $y = 0$, $x = 0$ and $x = 1$ about the line y=-1.



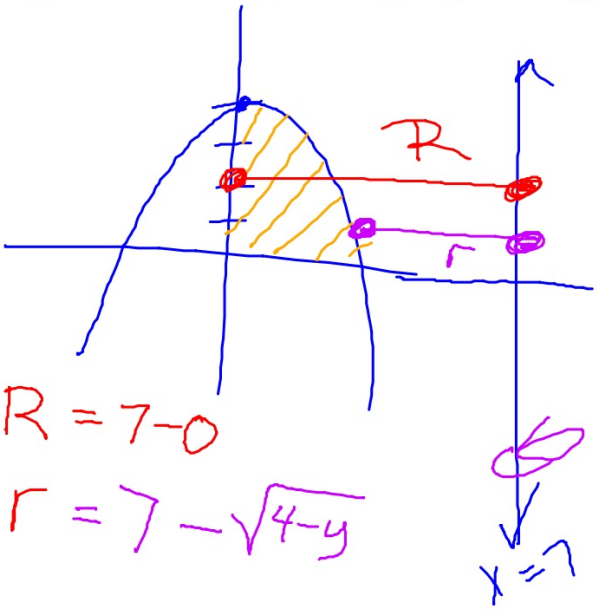
$$\pi \int_0^1 \left((e^x+1)^2 - (1)^2 \right) dx$$

(R^2, r^2)

$R(x) = e^x - (-1) = e^x + 1$

$r(x) = 0 - (-1) = 1$

ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded in in the first quadrant by the graph of $y = 4 - x^2$ about the line x=7.



$R = 7 - 0$

$r = 7 - \sqrt{4-y}$

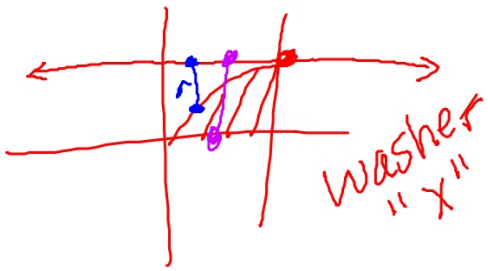
$$\pi \int_0^4 \left(7^2 - \left(7 - \sqrt{4-y}\right)^2\right) dy$$

ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$, $x = 9$ and the coordinate axes about the line...

a) x=9.

$$\pi \int_0^3 (y^2 - 9)^2 \, dy$$

b) y=3.



$$\pi \int_0^9 \left((3-0)^2 - (3-\sqrt{x})^2 \right) dx$$

ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$, $x = 9$ and the coordinate axes about the line...

a) x=9.

ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$, $x = 9$ and the coordinate axes about the line...

b) y=3.

ex: Set up an integral expression to find the volume of the solid formed by revolving the region bounded in in the third quadrant by the graphs of

$$y = x^2 - 5, \quad x = -\sqrt{5} \text{ and } y = -6$$

about the y-axis.