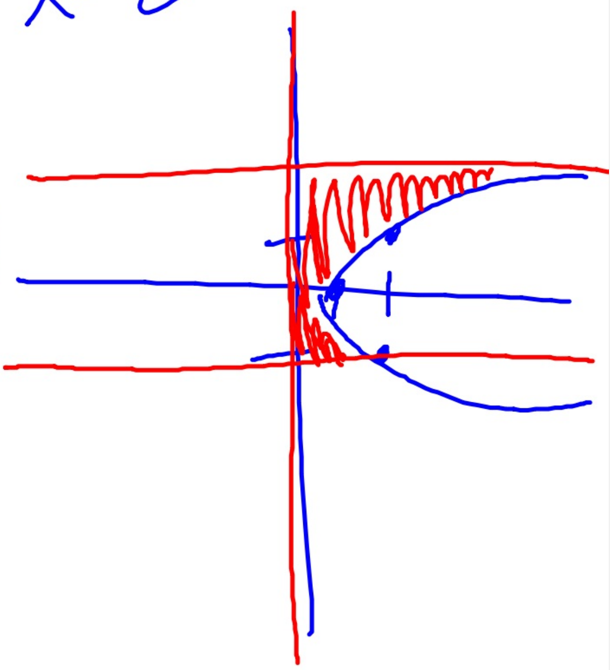
29.) $f(y) = y^2 + 1$ $\quad g(y) = 0$ $\quad y = -1, y = 2$

$x = y^2 + 1$ $\quad x = 0$

x	y
2	-1
1	0
2	1

$$\int_{-1}^{2} \left(y^2 + 1 - 0\right) dy$$



41.) $f(x)=2\sin x \quad g(x)=\tan x$

$$2\int_0^{\pi/3}(2\sin x-\tan x)\,dx$$

$$2\left[-2\cos x+\ln|\cos x|\right]_0^{\pi/3}$$

6.2: Volume by cross section

You will be given:

- base (determined by region enclosed by functions)
- geometric shape
- perpendicular to x-axis or y-axis

Perpendicular to the x-axis

$$V = \int_a^b (\text{Area})\,dx$$

in terms of x

Perpedicular to the y-axis

$$V = \int_a^b (\text{Area})\,dy$$

in terms of y

Geometric Shapes

Square s^2

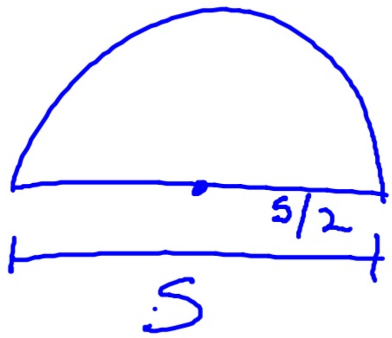
Rectangle $s \cdot h$

Equilateral triangle $\frac{\sqrt{3}}{4}s^2$

Right Isosceles Triangle(leg on base) $\frac{1}{2}s^2$

Right Isosceles Triangle (hyp. on base) $\frac{1}{4}s^2$

Semi-circles $\frac{\pi}{8}s^2$

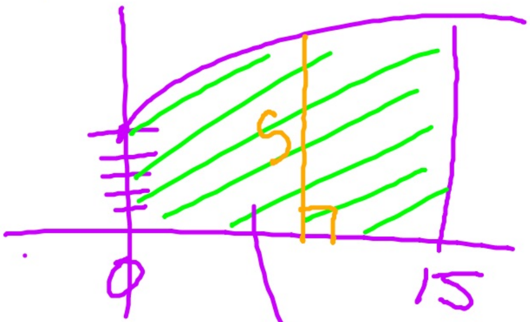


$\frac{1}{2}\pi r^2$

$\frac{1}{2}\pi\left(\frac{s}{2}\right)^2$

1 $f(x) = 2\sqrt{x} + 5$ $[0, 15]$, x-axis

Cross section perpendicular to the x-axis are squares.



$$\int_0^{15} S^2\,dx = \int_0^{15} \left(2\sqrt{x}+5\right)^2 dx$$

$$S = \left(2\sqrt{x}+5-0\right)$$

1599.597

② $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

Cross section perpendicular to the x-axis are semi-circles.

$S = (2\sqrt{x} + 5)$

$$\int_0^{15} \frac{\pi}{8} S^2 \, dx = \frac{\pi}{8} \int_0^{15} (2\sqrt{x} + 5)^2 \, dx$$

628.160

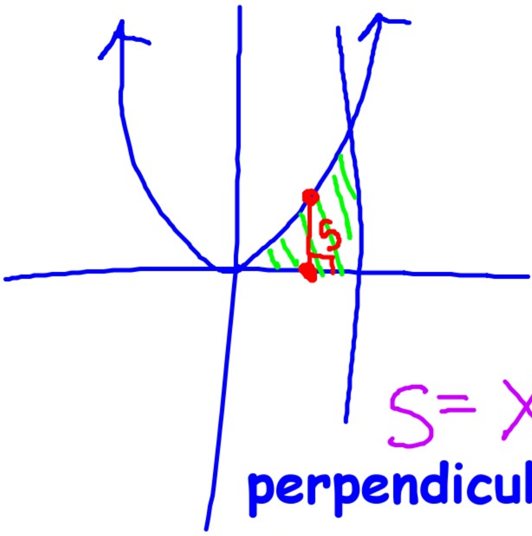
3 $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

Cross section perpendicular to the x-axis are rectangles of height 4.

$$\int_0^{15} 4s\,dx = \int_0^{15} 4(2\sqrt{x}+5)\,dx$$

$$609.839$$

4) Base enclosed by $y = x^2$, $y = 0$, and $x = 2$. Cross sections perpendicular to the x-axis are squares.



$$\int_0^2 s^2\,dx = \int_0^2 (x^2)^2\,dx$$

$$= \int_0^2 x^4\,dx$$

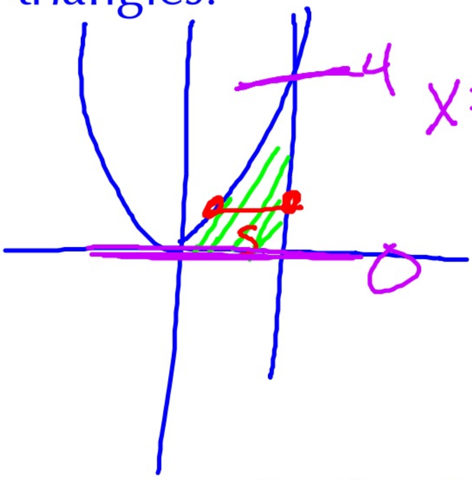
$s = x^2 - 0$

perpendicular to x-axis

S = top - bottom

6.4

5) Base enclosed by $y = x^2$, $y = 0$, and $x = 2$. Cross sections perpendicular to the y-axis are equilateral triangles. $\frac{\sqrt{3}}{4}s^2$



$x = \pm\sqrt{y}$

$$\int_0^4 \left(2-\sqrt{y}\right)^2 dy$$

1.155

perpendicular to y-axis:

S = right - left $= 2-\sqrt{y} = S$

ex: Find the volume of the solid whose base is bounded by $y = \ln x$, $x = e$, and the $x-$axis with...

$y = \ln x$
$e^y = x$

a) square cross sections taken perpendicular to thex-axis.

b) equilateral triangular cross sections taken perpendicular to the y-axis.

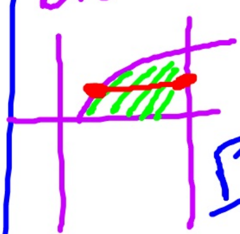
a.)

$s = \ln x$

$$\int_1^e (\ln x)^2 \, dx$$

0.718

b.)



$S = e - e^y$

$$\frac{\sqrt{3}}{4}\int_0^1 (e - e^y)^2 \, dy$$

.538