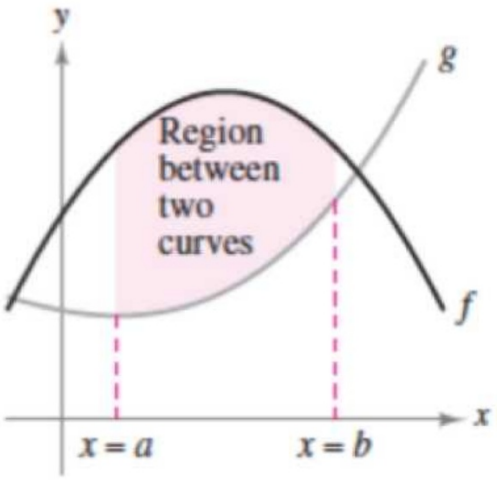
6.1 Area Between Two Curves

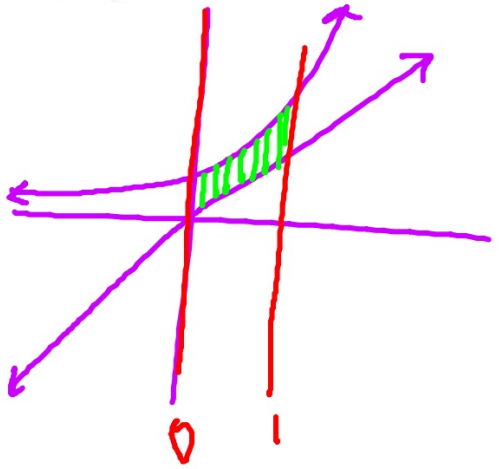
top − bottom



$$\text{Area} = \int_a^b (f(x) - g(x))\,dx$$

ex: Find the area enclosed by the given curves.

a) $y = e^x, \quad y = x, \quad x = 0, \quad x = 1$



$$A = \int_0^1 (e^x - x)\,dx$$

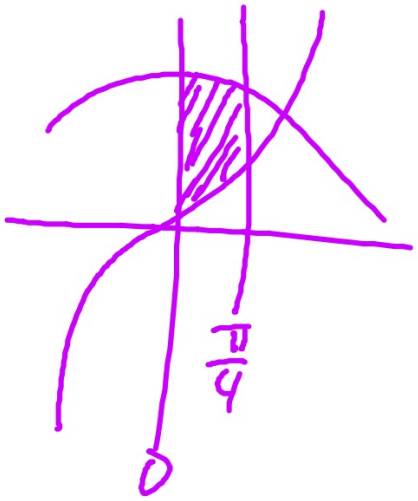
$$e^x - \frac{1}{2}x^2 \Big|_0^1$$

$$\left(e - \frac{1}{2}\right) - (1 - 0)$$

$$e - \frac{3}{2}$$

ex: Find the area enclosed by the given curves.

b) $y = \tan x, \quad y = 2\cos x, \quad 0 \le x \le \frac{\pi}{4}$



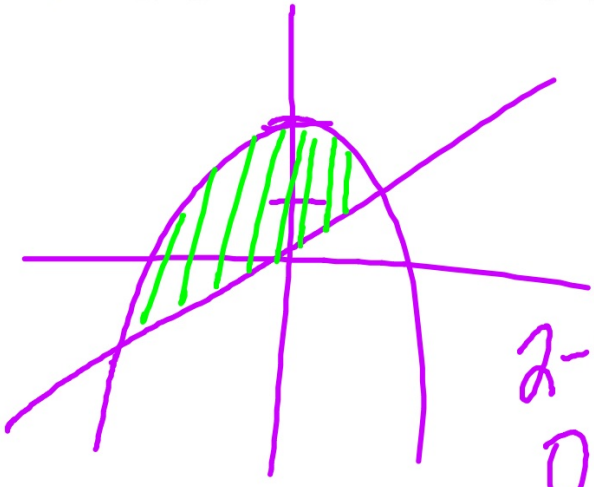
$$A = \int_0^{\pi/4} (2\cos x - \tan x)\,dx$$

$$= 2\sin x + \ln|\cos x| \Big|_0^{\pi/4}$$

$$\left(\sqrt{2} + \ln\frac{\sqrt{2}}{2}\right) - (\ 0\ )$$

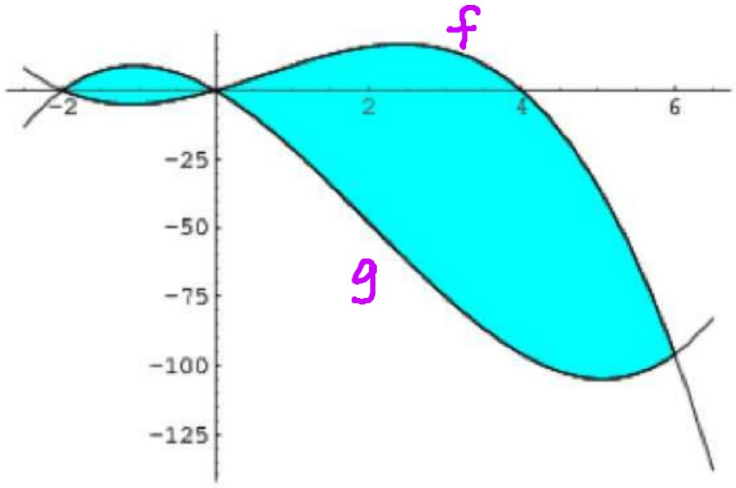
ex: Find the area enclosed by the given curves.

c) $f(x) = 2 - x^2, \quad g(x) = x$



$$A = \int_{-2}^{1} (2 - x^2 - x)\,dx = 4.5$$

$$2 - x^2 = x$$
$$0 = x^2 + x - 2$$
$$0 = (x+2)(x-1)$$
$$x = 1, -2$$



(by hand) Area = $\int_{-2}^{0}(g(x)-f(x))\,dx+\int_{0}^{6}(f(x)-g(x))\,dx$

OR

(calculator) Area = $\int_{-2}^{6}|f(x)-g(x)|\,dx$ OR $\int_{-2}^{6}|g(x)-f(x)|\,dx$

ex: Find the area enclosed by the given curves.

d) $f(x)=3x^3-x^2-10x, \quad g(x)=-x^2+x$

$$\int_{-1.915}^{1.915} |f(x)-g(x)|\,dx = 20.167$$

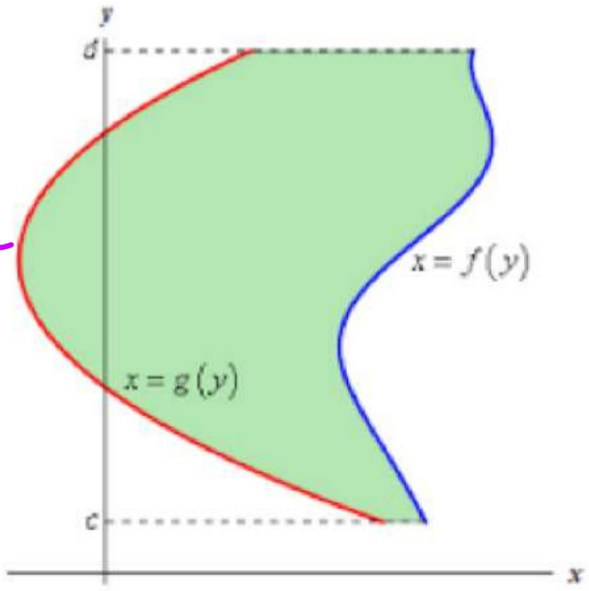
ex: Find the area enclosed by the given curves.

e) $y = \dfrac{1}{1+x^2}$, $y = \dfrac{x^2}{2}$

y_1 y_2

$$\int_{-1}^{1} (y_1 - y_2)\,dx = 1.237$$

Right − left



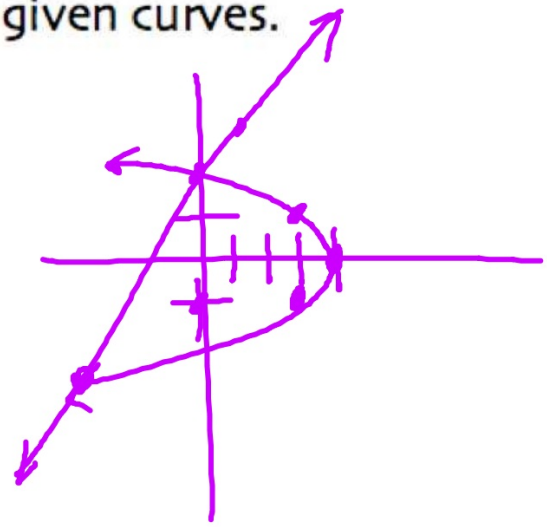
$$\text{Area} = \int_c^d (f(y) - g(y))\,dy$$

ex: Find the area enclosed by the given curves.

f) $x = 4 - y^2, \quad x = y - 2$

x	y
4	0
3	1
3	-1

x	y
0	2
1	3



$4 - y^2 = y - 2$

$y^2 + y - 6 = 0$

$(y+3)(y-2) = 0$

$$\int_{-3}^{2} \left(4 - y^2 - (y-2)\right) dy$$

20.833

ex:

a) Find the area bounded by

$$y = x, \quad y = -x^3 + 2, \quad \text{y-axis}$$

ex:

b) Find the area bounded by

$$y = x, \quad y = -x^3 + 2, \quad \text{x-axis}$$

ex: The line x=p divides the area bounded by $y = \sin x$ on $0 \le x \le \pi$ into 2 regions such that the area from $0 \le x \le p$ exceeds the area from $p \le x \le \pi$ by 1 square unit. Find p.

$$\int_0^p \sin x\,dx = \int_p^\pi \sin x\,dx + 1$$

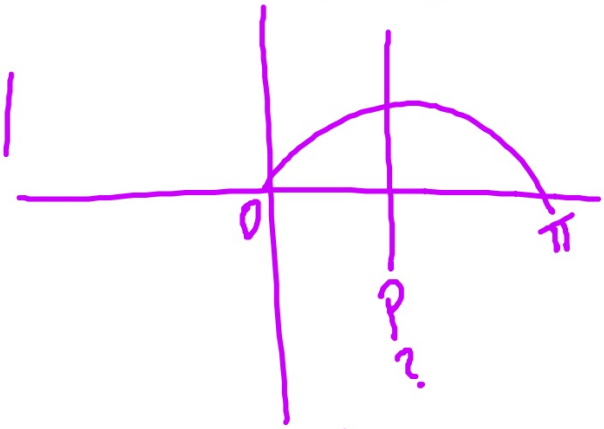
$$-\cos x \Big|_0^p = -\cos x \Big|_p^\pi + 1$$

$$-\cos p + 1 = -\cos\pi + \cos p + 1$$

$$-2\cos p = 1$$

$$\cos p = -\frac{1}{2}$$

$$p = \frac{2\pi}{3}$$

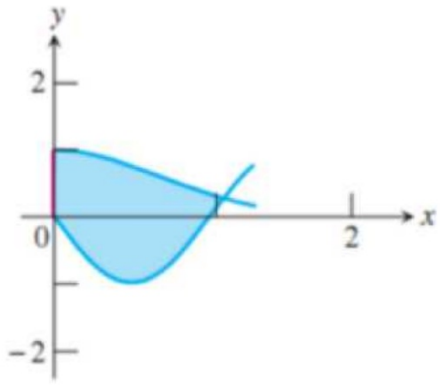


5.1, 5.3, 6.1 Extra Practice

2.

(Calculator permitted) Let R be the shaded region enclosed by the graphs of $y = e^{-x^2}$, $y = -\sin(3x)$, and the y-axis as shown at right. Which of the following gives the approximate area of the region R?

(A) 1.139 (B) 1.445 (C) 1.869 (D) 2.114 (E) 2.340



5.1, 5.3, 6.1 Extra Practice

5.

Let f and g be the functions given by $f(x)=e^x$ and $g(x)=\frac{1}{x}$. Which of the following gives the area of the region enclosed by the graphs of f and g between $x=1$ and $x=2$?

(A) $e^2-e-\ln 2$ (B) $\ln 2-e^2+e$ (C) $e^2-\frac{1}{2}$ (D) $e^2-e-\frac{1}{2}$ (E) $\frac{1}{e}-\ln 2$

5.1, 5.3, 6.1 Extra Practice

6.

Let R be the region in the first quadrant bounded by the x-axis, the graph of $x = y^2 + 2$, and the line $x = 4$. Which of the following integrals gives the area of R?

(A) $\int_0^{\sqrt{2}} \left[4 - \left(y^2 + 2\right)\right] dy$ (B) $\int_0^{\sqrt{2}} \left[\left(y^2 + 2\right) - 4\right] dy$ (C) $\int_{-\sqrt{2}}^{\sqrt{2}} \left[4 - \left(y^2 + 2\right)\right] dy$

(D) $\int_{-\sqrt{2}}^{\sqrt{2}} \left[\left(y^2 + 2\right) - 4\right] dy$ (E) $\int_2^4 \left[4 - \left(y^2 + 2\right)\right] dy$

5.1, 5.3, 6.1 Extra Practice

7.

Which of the following gives the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 3$.

(A) 2 (B) $\frac{9}{2}$ (C) $\frac{13}{2}$ (D) 13 (E) $\frac{27}{2}$

5.1, 5.3, 6.1 Extra Practice

11.

What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$