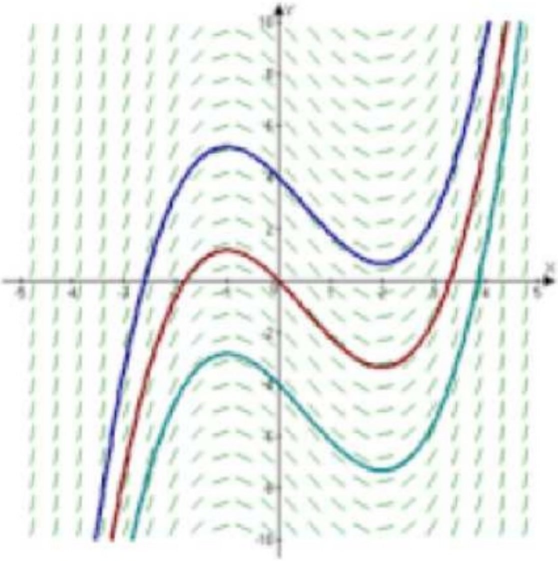
5.1/5.3 Slope Fields & Differential Equations

What is a Slope Field?



*See printout.

5.1/5.3 Slope Fields & Solving Differential Equations - Notes

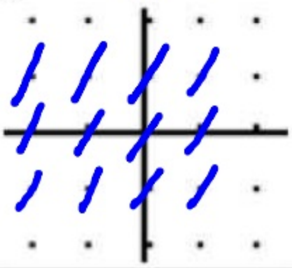
- What is a Slope Field? possible slope values for a differential equation

- Sketching a Slope Field

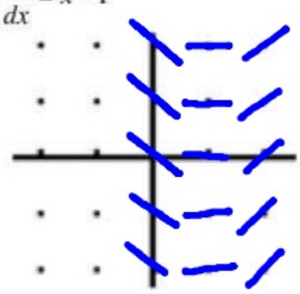
1. calculate slopes
2. draw slope segments

Ex 1: Sketch each slope field.

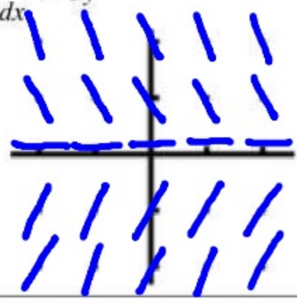
a) $\frac{dy}{dx} = 2$ $y = 2x + C$



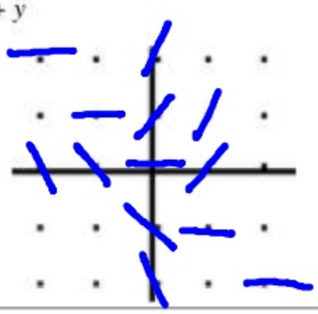
b) $\frac{dy}{dx} = x - 1$



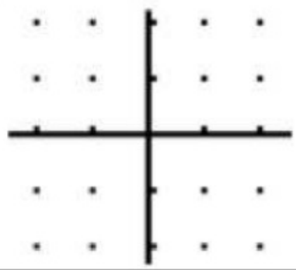
c) $\frac{dy}{dx} = -3y$



d) $\frac{dy}{dx} = x + y$

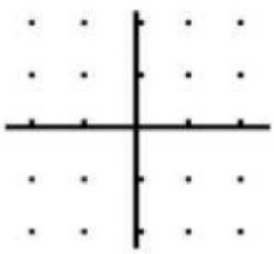


e) $\frac{dy}{dx} = y + xy$



Ex 2: Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

a) Sketch a slope field.

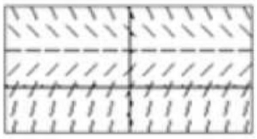


b) On the slope field above, sketch a solution curve that passes through the point (0, 1).

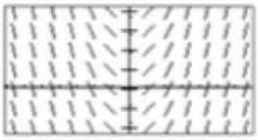
c) On the slope field above, sketch a solution curve that passes through the point (0, -1).

Ex 3: Match each differential equation with the slope field.

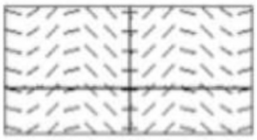
(A)



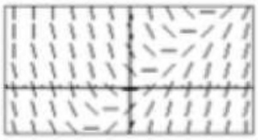
(B)



(C)



(D)



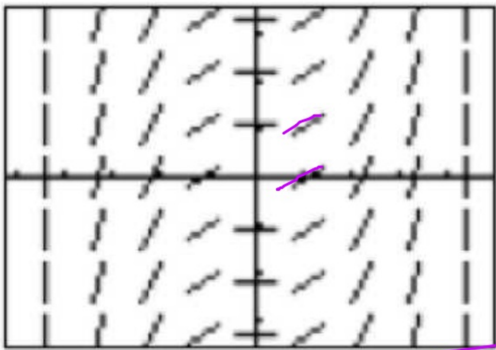
I. $\frac{dy}{dx} = \sin x$ C

II. $\frac{dy}{dx} = x - y$ D

III. $\frac{dy}{dx} = 2 - y$ A

IV. $\frac{dy}{dx} = x$ B

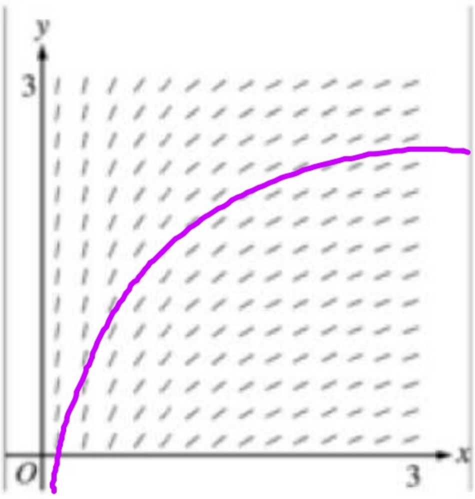
Ex 4: The slope field for a certain differential equation is shown below. Which of the following could be a particular solution to the differential equation?



$y' = \frac{1}{2}x^2$

(a) $y = \sin x$ (b) $y = \cos x$ (c) $y = x^2$ (d) $y = \frac{1}{6}x^3$ (e) $y = \ln x$

Ex 5: The slope field for a certain differential equation is shown below. Which of the following could be a particular solution to the differential equation?



(a) $y = x^2$ (b) $y = e^x$ (c) $y = e^{-x}$ (d) $y = \cos x$ (e) $y = \ln x$

Ex 5: Verify the solution of the differential equation.

a)

Solution	Differential Equation
$y = e^{-2x}$	$3y' + 5y = -e^{-2x}$

$y' = -2e^{-2x}$

$3(-2e^{-2x}) + 5e^{-2x} = -e^{-2x}$

$-e^{-2x} = -e^{-2x}$ ✓

Ex 5: Verify the solution of the differential equation.

b)

Solution	Differential Equation
$y = 3\cos x + \sin x$	$y'' + y = 0$

$y'' = -3\cos x - \sin x$

$-3\cos x - \sin x + 3\cos x + \sin x = 0$

$0 = 0$ ✓

Two Types of Solutions to Differential Equations

1. general (+C)

2. particular

Ex 7: Find the general solution.

a) $y' = \frac{2x}{y}$

$$y \cdot \frac{dy}{dx} = \frac{2x}{\cancel{y}} \cdot \cancel{y}$$

$$\cancel{x}\frac{y\,dy}{\cancel{dx}} = 2x\,dx$$

$$\int y\,dy = \int 2x\,dx$$

$$\frac{1}{2}y^2 + C_1 = x^2 + C_2$$

$$\frac{1}{2}y^2 = x^2 + C$$

$$y^2 = 2x^2 + C$$

$$y = \pm\sqrt{2x^2 + C}$$

Ex 7: Find the general solution.

b) $y' = 3y$

$$\frac{dy}{dx} = 3y$$

$$\frac{dy}{y} = 3dx$$

$$\ln|y| = (3x + C)$$

$$e^{\ln|y|} = e^{3x+C}$$

$$|y| = e^{3x+c}$$

$$|y| = e^{3x} \cdot e^{c}$$

$$e^{c} \to c$$

$$|y| = Ce^{3x}$$

$$y = Ce^{3x}$$

Ex 8: Find the particular solution.

a) $y' = 7y$, (10, 1)

$y = Ce^{7x}$

$1 = Ce^{70}$

$e^{-70} = C$

$y = e^{-70} \cdot e^{7x}$

$\boxed{y = e^{7x-70}}$

or

$\frac{e^{7x}}{e^{70}}$

Ex 8: Find the particular solution.

b) $y' = \dfrac{x}{y}$, (0, -1)

$$\frac{dy}{dx} = \frac{x}{y}$$

$$y\,dy = x\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$$

$$\frac{1}{2} = C$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + \frac{1}{2}$$

$$\sqrt{y^2} = \sqrt{x^2 + 1}$$

$$y = \pm\sqrt{x^2+1}$$

$$y = -\sqrt{x^2+1}$$

Ex 8: Find the particular solution.

c) $y' = \dfrac{y}{x^2}$, (1, 3)

$$\frac{dy}{\cancel{dx}} = \frac{\cancel{y}\,dx}{x^2}$$

$$\frac{dy}{y} = \frac{dx}{x^2}$$

$$\ln|y| = -\frac{1}{x} + C$$

$$e$$

$$|y| = e^{-1/x + c}$$

$$|y| = e^{-1/x} \cdot e^{c}$$

$$|y| = Ce^{-1/x}$$

$$3 = Ce^{-1}$$

$$3e = C \checkmark$$

$$y = 3e \cdot e^{-1/x}$$

Ex 9: The rate of change of y with respect to x is proportional to the difference between x and 4. Write a differential equation.

$$\frac{dy}{dx} = K(x-4)$$

Ex 10: The rate of change of y with respect to x varies directly with the square of y. Write a differential equation.

$$\frac{dy}{dx} = ky^2$$

K: constant of proportionality

The rate of change of y is proportinal to y.

$$\frac{dy}{dx} = ky$$

$$\frac{dy}{y} = k\,dx$$

$$e^{\ln|y|} = e^{kx+C}$$

$$|y| = Ce^{kx}$$

b) negative slopes when y < 2 but x is not 0.

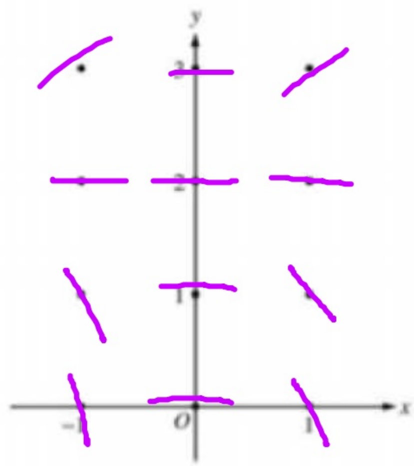
Ex 14:

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the *xy*-plane. Describe all points in the *xy*-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.



$$\frac{dy}{y-2} = x^4\,dx$$

$$e^{\ln|y-2|} = e^{\frac{x^5}{5} + C}$$

$$|y-2| = Ce^{x^5/5}$$

$$2 = C$$

$$\cancel{|y-2| = 2e^{x^5/5}}$$

$$y = -2e^{x^5/5} + 2$$