$$f''(x) = \frac{2}{x^2} = 2x^{-2}$$

$$\boxed{f'(1) = 1 \qquad f(1) = 1}$$

$$f'(x) = \frac{2 \cdot x^{-1}}{-1} + C$$

85.) $\int_1^x \frac{3}{t}\,dt = \int_{1/4}^x \frac{1}{t}\,dt$

$3\ln|t| \Big|_1^x = \ln|t| \Big|_{1/4}^x$

$3(\ln x - 0) = \ln x - \ln\frac{1}{4}$

$3\ln x - \ln x = -\ln\frac{1}{4}$ $\quad \ln\left(\frac{1}{4}\right)^{-1}$

$2\ln x = \ln 4$ $\quad x^2 = 4$ $\quad \cancel{x=2}$

$e^{\ln x^2} = e^{\ln 4}$

41.) $\int e^{-x}\tan(e^{-x})\,dx$

$u = e^{-x}$

$du = -e^{-x}\,dx$

$\frac{du}{-e^{-x}} = dx$

$\int \cancel{e^{-x}}\tan u \cdot \frac{du}{-\cancel{e^{-x}}}$

$-\int \tan u\,du$

$-\left(-\ln|\cos(e^{-x})|\right) + C$

13. $\displaystyle\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx = \frac{1}{3}\int \frac{1}{u}\,du$

$= \frac{1}{3}\ln\left|x^3+3x^2+9x\right| + C$

$u = x^3+3x^2+9x$

$du = (3x^2+6x+9)\,dx$

$du = 3(x^2+2x+3)\,dx$

$$\int \cot \frac{\theta}{3}\, d\theta = 3\int \cot u\, du$$

$u = \frac{\theta}{3}$

$du = \frac{1}{3} d\theta$

$$= 3 \ln\left|\sin \frac{\theta}{3}\right| + C$$

4.7 Inverse Trigonometry: Integration

Review:

$$\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\csc^{-1} x\right] = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left[\cos^{-1} x\right] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\cot^{-1} x\right] = \frac{-1}{1+x^2}$$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$u = 1 - x^2$$

$$du = -2x\,dx$$

$$\frac{du}{-2x} = dx$$

$$-\frac{1}{2}\int u^{-1/2}\,du$$

$$-\frac{1}{2} \cdot \frac{u^{1/2}}{1/2} + C$$

$$-\sqrt{1-x^2} + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\arcsin x + C$$

$$\int \frac{1}{1+x^2}\,dx$$

$\arctan x + C$

$$\int \frac{x}{1+x^2}\,dx$$

$\frac{1}{2}\ln|1+x^2| + C$

THEOREM 4.20 Integrals Involving Inverse Trigonometric Functions

Let u be a differentiable function of x, and let $a > 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$

2. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

3. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

ex: Integrate.

a) $\int \frac{dx}{1+9x^2}$ $= \frac{1}{3} \cdot \frac{1}{1} \arctan \frac{3x}{1} + C$

$u = 3x$

$du = 3dx$

$a = 1$

$\frac{du}{3} = dx$

$\frac{1}{3}(\arctan 3x) + C$

Check: $\frac{1}{3}\left[\frac{3}{1+9x^2}\right]$ ✓

$$\int \frac{x}{\sqrt{16-x^4}}\,dx$$

$u = x^2$

$du = 2x\,dx$

$a = 4$

$$\frac{1}{2}\cdot \arcsin\frac{x^2}{4} + C$$

$$\int_0^{\sqrt{2}/2} \frac{\arccos x}{\sqrt{1-x^2}}\,dx$$

$$\int_0^{\sqrt{2}/2} \arccos x \cdot \frac{1}{\sqrt{1-x^2}}\,dx$$

$u = \arccos x \quad du = -\frac{1}{\sqrt{1-x^2}}\,dx$

$$-\int u\,du = -\frac{u^2}{2} = -\frac{(\arccos x)^2}{2}\Bigg|_0^{\sqrt{2}/2}$$

$$-\left(\frac{\left(\frac{\pi}{4}\right)^2}{2} - \frac{\left(\frac{\pi}{2}\right)^2}{2}\right)$$

$$-\left(\frac{\pi^2}{32} - \frac{\pi^2}{8}\right)$$

$$\frac{3\pi^2}{32}$$

ex: Integrate.

1) $\int \frac{dx}{x^2-2x+2} = \int \frac{dx}{(x-1)^2+1} = \arctan\frac{x-1}{1} + C$

x^2-2x+2

$x^2-2x+1+2-1$

$(x-1)^2+1$

$u = x-1$

$du = 1dx$

$a = 1$

ex: Integrate.

n) $\int \frac{dx}{\sqrt{-x^2-4x}} = \int \frac{dx}{\sqrt{4-(x+2)^2}} = \arcsin \frac{x+2}{2} + C$

$u = x+2$

$du = 1dx$

$a = 2$

$-(x^2+4x+4)+4$

$-(x+2)^2+4$

$4-(x+2)^2$

ex: Integrate.

o) $\int \frac{dx}{x^2+4x+13}$ $= \int \frac{dx}{(x+2)^2+9}$

$x^2+4x+4+13-4$

$(x+2)^2+9$

$u = x+2$

$du = 1dx$

$a = 3$

$\frac{1}{3}\arctan\frac{x+2}{3} + C$

ex: Integrate.

p) $\int \frac{x+5}{\sqrt{9-x^2}}dx = \int \frac{x}{\sqrt{9-x^2}}dx + \int \frac{5}{\sqrt{9-x^2}}dx$

$u = 9 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}\int u^{-1/2}\,du$

$-\frac{1}{2}\cdot\frac{u^{1/2}}{1/2} + C$

$-\sqrt{9-x^2} + 5\arcsin\frac{x}{3} + C$

ex: Integrate.

$u = x$
$du = 1\,dx$
$a = 3$

q) $\int \frac{x+1}{x^2+9}dx = \int \frac{x}{x^2+9}dx + \int \frac{1}{x^2+9}dx$

$\frac{1}{2}\ln|x^2+9| + \frac{1}{3}\arctan\frac{x}{3} + C$