81.) $\displaystyle\int_1^3 \frac{e^{3/x}}{x^2}\,dx$

$u = \frac{3}{x}$

$du = \frac{-3}{x^2}\,dx$

$$-\frac{1}{3}\int_3^1 e^u\,du = \frac{1}{3}\int_1^3 e^u\,du$$

$e^{3/x}\left(\frac{1}{x^2}\right)dx$

$$= \frac{1}{3}e^u \Big|_1^3 = \frac{1}{3}\left(e^3 - e\right)$$

49.) $f''(x) = \frac{2}{x^2}$ $f'(1)=1$ $f(1)=1$

$f'(x) = -\frac{2}{x} + C$

$1 = -2 + C$

$3 = C$

$f'(x) = -\frac{2}{x} + 3$

$f(x) = \underbrace{-2\ln|x|} + 3x + C$

$1 = 0 + 3 + C$; $C = -2$

$f(x) = -2\ln|x| + 3x - 2$

57.) $\int_1^2 \frac{1-\cos\theta}{\theta-\sin\theta}\,d\theta$

$u=\theta-\sin\theta$

$du=1-\cos\theta\,d\theta$

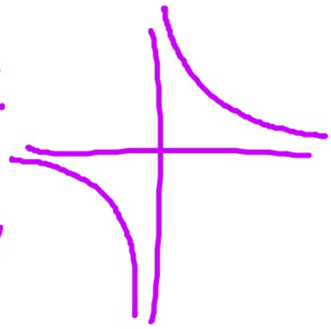
$\int \frac{1}{u}\,du = \ln|\theta-\sin\theta|\Big|_1^2$

$\ln|2-\sin 2| - \ln|1-\sin 1|$

$\ln\frac{2-\sin 2}{1-\sin 1}$

85.) $\int_{1}^{2x} \frac{3}{t}\,dt = \int_{1/4}^{2x} \frac{1}{t}\,dt$

$3\ln|t| \Big|_{1}^{x} = \ln|t| \Big|_{1/4}^{x}$



$3\ln|x| = \ln|x| - \ln\frac{1}{4}$

$2\ln|x| = -\ln\frac{1}{4}$

$e^{\ln|x^2|} = e^{\ln 4}$

$x^2 = 4$

$\cancel{x = \pm 2}$

$x = 2$

13.) $\int_0^4 \frac{2x}{x^2+9}\,dx$

4. C

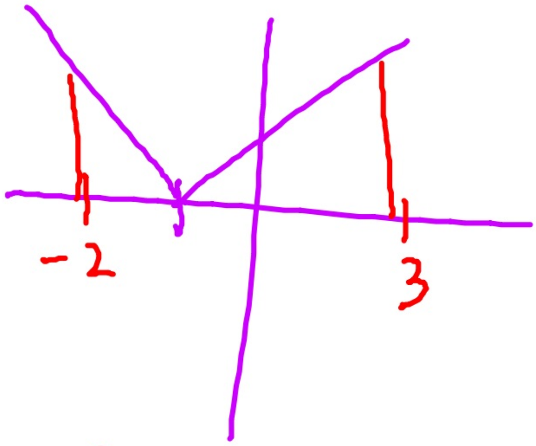
14. A

$u = x^2 + 9$

$du = \underline{\underline{2x}}\,dx$

$\int_9^{25} \frac{1}{u}\,du = \ln|u| \Big|_9^{25} = \ln 25 - \ln 9$

8.) $\int_{-2}^{+3} |x+1|\,dx$



$\int_{0}^{3} \sqrt{9-x^2}\,dx$

4.6 Continued

Trigonometric Antiderivatives

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \tan x\,dx = -\ln|\cos x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

ex: Integrate.

a) $\int \tan 5x\, dx = \frac{1}{5}\int \tan u\, du$

$u = 5x$

$du = 5\, dx$

$= -\frac{1}{5}\ln|\cos 5x| + C$

Check:

$\checkmark\ -\frac{1}{5}\left(\frac{-\sin 5x \cdot 5}{\cos 5x}\right)$

ex: Integrate.

b) $\int_{\pi/4}^{\pi/2} \sqrt{\csc^2 x - 1}\, dx$ $= \int_{\pi/4}^{\pi/2} |\cot x|\, dx = \ln|\sin x| \Big|_{\pi/4}^{\pi/2}$

$0 - \ln\frac{\sqrt{2}}{2}$

$\ln\frac{2}{\sqrt{2}}$

$\ln\sqrt{2}$

$\frac{1}{2}\ln 2$

ex:

Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \frac{2e^{\tan x}+5}{\cos^2 x}\,dx$

(A) $2e+3$ (B) $2e$ (C) $2e-3$ (D) e (E) $e+5$

ex:

Evaluate $\int_{e}^{e^4} \frac{5}{x\sqrt{\ln x}}\,dx$

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

FR 7

A particle starts at the point $(5,0)$ at $t=0$ and moves along the x-axis in such a way that at time $t>0$ its velocity $v(t)$ is given by $v(t)=\dfrac{t}{1+t^2}$.

(a) Determine the maximum velocity attained by the particle. Justify your answer.

(b) Determine the position of the particle at $t=6$.

(c) Find the limiting value of the velocity as t increases without bound.

$$\int \sec\left(\frac{x}{2}\right)dx = 2\ln\left|\sec\frac{x}{2}+\tan\frac{x}{2}\right|+C$$

4.7 Inverse Trigonometry: Integration

Review:

$$\frac{d}{dx}\left[\sin^{-1}x\right] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\csc^{-1}x\right] = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left[\cos^{-1}x\right] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\sec^{-1}x\right] = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left[\tan^{-1}x\right] = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\cot^{-1}x\right] = \frac{-1}{1+x^2}$$

THEOREM 4.20 Integrals Involving Inverse Trigonometric Functions

Let u be a differentiable function of x, and let $a > 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$
2. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$
3. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

ex: Integrate.

a) $\int \frac{dx}{1+9x^2} = \frac{1}{3} \arctan 3x + C$

$u = 3x$

$du = 3dx$

$a = 1$

check:

$\cancel{\frac{1}{3}}\left(\frac{\cancel{3}}{1+9x^2}\right)$ ✓

ex: Integrate.

b) $\int \frac{2\,dx}{2x\sqrt{4x^2-9}} = \frac{1}{3}\operatorname{arcsec}\frac{|2x|}{3} + C$

$u = 2x$

$du = 2\,dx$

$a = 3$

ex: Integrate.

c) $\int \frac{\sin x dx}{\sqrt{25-\cos^2 x}} = -\arcsin\frac{\cos x}{5} + C$

$u = \cos x$

$du = -\sin x dx$

$a = 5$

ex: Integrate.

d) $7\int_0^4 \frac{dx}{16+x^2} = 7 \cdot \frac{1}{4} \arctan \frac{x}{4} \Big|_0^4$

$u = x$

$du = 1dx$

$a = 4$

$\frac{7}{4}(\arctan 1 - \arctan 0)$

$\frac{7}{4}\left(\frac{\pi}{4} - 0\right) = \frac{7\pi}{16}$

ex: Integrate.

e) $\int \frac{xdx}{\sqrt{25-x^2}}$

u-sub

ex: Integrate.

g) $\int \frac{e^{2x}dx}{1+e^{2x}}$ $= \frac{1}{2}\int \frac{1}{u}du$

$u = 1+e^{2x}$

$du = 2e^{2x}dx$

$\frac{1}{2}\ln|1+e^{2x}|+c$

ex: Integrate.

h) $\int \frac{\sin x dx}{1+\cos^2 x}$

ex: Integrate.

k) $\int \frac{\arccos x dx}{\sqrt{1-x^2}}$ $= -\int u\,du$

$u = \arccos x$

$du = \frac{-1}{\sqrt{1-x^2}} dx$

$\int \arccos x \cdot \frac{1}{\sqrt{1-x^2}} dx$

$-\frac{u^2}{2} + C$

$-\frac{(\arccos x)^2}{2} + C$

ex: Integrate.

1) $\int \frac{dx}{x^2 - 2x + 2}$ $= \int \frac{dx}{(x-1)^2+1}$

$x^2 - 2x + 1 \quad + 2 - 1$

$(x-1)^2 + 1$

CTS

$u = x - 1$

$du = 1dx$

$a = 1$

$\frac{1}{1}\arctan\frac{x-1}{1} + C$

ex: Integrate.

n) $\int \frac{dx}{\sqrt{\underline{-x}^2 - 4x}} = \int \frac{dx}{\sqrt{4-(x+2)^2}}$

$-(x^2+4x \underline{+4}) \underline{+4}$

$4-(x+2)^2$

$\arcsin \frac{x+2}{2} + C$

ex: Integrate.

o) $\int \frac{dx}{x^2+4x+13}$

ex: Integrate.

p) $\int \frac{x+5}{\sqrt{9-x^2}}dx$

ex: Integrate.

q) $\int \frac{x+1}{x^2+9}dx = \int \frac{x}{x^2+9}dx + \int \frac{1}{x^2+9}dx$

$\frac{1}{2}\ln|x^2+9| + \frac{1}{3}\arctan\frac{x}{3} + C$

ex: Integrate.

s) $\int \frac{2x-3}{\sqrt{4x-x^2}}dx$

ex: Integrate.

☆ t) $\int \frac{x^3}{1+x^2}dx$