

4.6 The Natural Logarithmic Function: Integration

Review:

$$\frac{d}{dx} [\ln x] =$$

$$\frac{d}{dx} [\ln|x|] =$$

$$\frac{d}{dx} [\ln u] =$$

THEOREM 4.19 Log Rule for Integration

Let u be a differentiable function of x .

$$1. \int \frac{1}{x} dx =$$

$$2. \int \frac{1}{u} du =$$

ex: Integrate.

a) $\int \frac{6}{x} dx$

ex: Integrate.

b) $\int \frac{1}{3x+5} dx$

ex: Integrate.

$$\textcircled{c} \int \frac{2x}{x^2 + 6} dx$$

On your own...

ex: Integrate.

d) $\int \frac{\sec^2 x}{\tan x} dx$

e) $\int \frac{\sec^2 x}{\tan^2 x} dx$

f) $\int \tan x dx$

g) $\int \frac{x}{\sqrt{9-x^2}} dx$

h) $\int \frac{1}{x \ln x} dx$

i) $\int \frac{x^3 + 4x^2 - 2x + 1}{x^2} dx$

ex: Integrate.

d) $\int \frac{\sec^2 x}{\tan x} dx$

ex: Integrate.

e) $\int \frac{\sec^2 x}{\tan^2 x} dx$

ex: Integrate.

f) $\int \tan x \, dx$

ex: Integrate.

$$g) \int \frac{x}{\sqrt{9-x^2}} dx$$

ex: Integrate.

h) $\int \frac{1}{x \ln x} dx$

ex: Integrate.

i) $\int \frac{x^3 + 4x^2 - 2x + 1}{x^2} dx$

ex: Integrate.

★) $\int \frac{x^2 + x + 1}{x^2 + 1} dx$

ex: Integrate.

★ ★ k) $\int \frac{2x}{(x+1)^2} dx$

ex: Integrate.

D) $\int_0^1 \frac{x-1}{x+1} dx$

ex: Integrate.

★ ★ m) $\int \frac{1}{1 + \sqrt{x}} dx$

Trigonometric Antiderivatives

$$\int \sin x \, dx =$$

$$\int \cos x \, dx =$$

$$\int \tan x \, dx =$$

$$\int \csc x \, dx =$$

$$\int \sec x \, dx =$$

$$\int \cot x \, dx =$$

ex: Integrate.

a) $\int \tan 5x \, dx$

ex: Integrate.

b) $\int_{\pi/4}^{\pi/2} \sqrt{\csc^2 x - 1} dx$

ex:

Evaluate $\int_0^{\frac{\pi}{4}} \frac{2e^{\tan x} + 5}{\cos^2 x} dx$

- (A) $2e + 3$ (B) $2e$ (C) $2e - 3$ (D) e (E) $e + 5$

ex:

Evaluate $\int_{e^4}^{e^4} \frac{5}{x\sqrt{\ln x}} dx$

- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

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A particle starts at the point $(5, 0)$ at $t = 0$ and moves along the x -axis in such a way that at time $t > 0$ its velocity $v(t)$ is given by $v(t) = \frac{t}{1+t^2}$.

- (a) Determine the maximum velocity attained by the particle. Justify your answer.
- (b) Determine the position of the particle at $t = 6$.
- (c) Find the limiting value of the velocity as t increases without bound.

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t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	-20	-30	-20	-14	-10	0	10
$a(t)$ (ft/sec ²)	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \leq t \leq 60$ seconds, the car's velocity v , measured in feet per second, and acceleration a , measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

- (a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)| dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)| dt$ using a trapezoidal approximation with the three subintervals determined by the table.
- (b) Using appropriate units, explain the meaning of $\int_0^{30} a(t) dt$ in terms of the car's motion. Find the exact value of $\int_0^{30} a(t) dt$.