4.5 U-Substitution

U-Substitution is an integration technique used when an integrand involves a composite function.

$$\boxed{f\left(g(x)\right)}$$

Review:

$$\frac{d}{dx}\left[f\left(g(x)\right)\right]=$$

THEOREM 4.15 Antidifferentiation of a Composite Function

Let g be a function whose range is an interval I, and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I, then

$$\int f'(g(x))g'(x)\,dx =$$

Letting $u = g(x)$ gives $du = g'(x)\,dx$ and

$$\int f'(f(u))\,du =$$

Evaluate.

a) $\int 2x\left(1+x^2\right)^5 dx$

ex: Evaluate.

b)

$$\int x\left(3x^2-5\right)^{12}dx$$

ex: Evaluate.

c)

$$\int 5\sin(7x)\,dx$$

ex: Evaluate.

d) $\int \tan^2 x \sec^2 x \, dx$

ex: Evaluate.

e)

$$\int (x+1)e^{x^2+2x}\,dx$$

ex: Evaluate.

f)

$$\int \frac{e^{2x} + 2e^{x} + 5}{e^{x}}\,dx$$

ex: Evaluate.

g)

$$\int(\cos 2x-\sin 3x)\,dx$$

ex: Evaluate.

h)

$$\int \frac{x}{\sqrt[3]{1-2x^2}}\,dx$$

ex: Evaluate.

i)

$$\int \cos^4(x)\sin(x)\,dx$$

ex: Evaluate.

j)

$$\int x\sqrt{x+1}\,dx$$

ex:

$\int e^x \cos(e^x + 1)\, dx =$

(A) $\sin(e^x + 1) + C$

(B) $e^x \sin(e^x + 1) + C$

(C) $e^x \sin(e^x + x) + C$

(D) $\frac{1}{2}\cos^2(e^x + 1) + C$

FR 1

A particle moves along the x-axis in such a way that its acceleration at time t for $t \geq 0$ is given by $a(t) = 4\cos(2t)$. At time $t = 0$, the velocity of the particle is $v(0) = 1$ and its position is $x(0) = 0$.

(a) Write an equation for the velocity $v(t)$ of the particle.

(b) Write an equation for the position $x(t)$ of the particle.

(c) For what values of t, $0 \leq t \leq \pi$, is the particle at rest?