

$$\begin{aligned}
 15) \quad & \int 5x(1-x^2)^{1/3} dx \\
 & 5 \int x(1-x^2)^{1/3} dx = 5 \int x \cdot u^{1/3} \cdot \frac{du}{-2x} \\
 u &= 1-x^2 \\
 du &= -2x dx \\
 \frac{du}{-2x} &= dx
 \end{aligned}$$

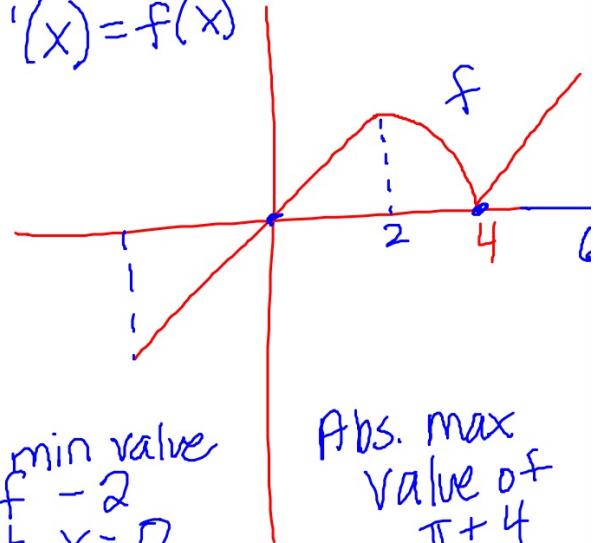
$$\begin{aligned}
 & -\frac{5}{2} \cdot \frac{u^{4/3}}{4/3} + C \\
 & -\frac{5}{2} \cdot \frac{3}{4} (1-x^2)^{4/3} + C \\
 & -\frac{15}{8} (1-x^2)^{4/3} + C
 \end{aligned}$$

1d.

$$F(x) = \int_{-2}^x f(t) dt$$

X	F(x)
-2	$\int_{-2}^{-2} f(t) dt = 0$
0	-2
6	$\int_2^6 f(t) dt = \pi + 4$

$$F'(x) = f(x)$$



Abs. min value
of f
at $x=0$

Abs. max
value of f
at $x=6$

$$3b.) \quad g(x) = \int_{-1}^x \sqrt{t^3 + 1} dt$$

$$g'(x) = \sqrt{x^3 + 1} \cdot 1$$

4.5 U-Substitution

U-Substitution is an integration technique used when an integrand involves a **composite function**.

$$f(g(x))$$

Review:

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

THEOREM 4.15 Antidifferentiation of a Composite Function

Let g be a function whose range is an interval I , and let f be a function that is continuous on I . If g is differentiable on its domain and F is an antiderivative of f on I , then

$$\int \underbrace{f'(g(x))}_{\text{Chain Rule}} g'(x) dx = F(g(x)) + C$$

ex: Evaluate.

b) $\int 4\cos(4x-3)dx$ = $\int \cos u \cdot \frac{du}{4}$

$u = 4x - 3$

$du = 4dx$

$\frac{du}{4} = dx$

$\sin u + C$

$\sin(4x-3) + C$

ex: Evaluate.

c) $\int \sec x \tan x \sin(\sec x) dx$

ex: Evaluate.

d) $\int \tan^2 x \sec^2 x dx$

ex: Evaluate.

$$\text{e) } \int x(3x^2 - 5)^{12} dx = \int x(u)^{12} \frac{du}{6x}$$

$$u = 3x^2 - 5$$

$$du = 6x dx$$

$$\frac{du}{6x} = dx$$

$$\begin{aligned} \frac{1}{6} \int u^{12} du &= \frac{1}{6} \cdot \frac{u^{13}}{13} + C \\ &= \frac{(3x^2 - 5)^{13}}{78} + C \end{aligned}$$

ex: Evaluate.

$$\text{f) } \int 5 \sin(7x) dx = \left\{ 5 \sin u \cdot \frac{du}{7} \right.$$
$$u = 7x$$
$$du = 7dx$$
$$\frac{du}{7} = dx$$
$$\left. -\frac{5}{7} \cos u + C \right.$$
$$\left. -\frac{5}{7} \cos(7x) + C \right.$$

ex: Evaluate.

$$\text{g) } \int (x+1)e^{x^2+2x} dx = \int (x+1)e^u \cdot \frac{du}{2(x+1)} \quad \left. \begin{array}{l} u = x^2 + 2x \\ du = (2x+2)dx \\ du = 2(x+1)dx \\ \frac{du}{2(x+1)} = dx \end{array} \right\} e^{5x} dx$$
$$\frac{1}{2} \int e^u du \quad \left. \begin{array}{l} u = 5x \\ du = 5dx \\ \frac{du}{5} = dx \end{array} \right\}$$
$$\frac{1}{2} e^u + C \quad \left. \begin{array}{l} \frac{1}{5} \int e^u du \\ \frac{1}{5} e^u + C \end{array} \right\}$$
$$\frac{1}{2} e^{x^2+2x} + C$$

ex: Evaluate.

$$\text{h) } \int (x^2 - 3 + \sec^2(5x)) dx$$

$$\frac{1}{3}x^3 - 3x + \frac{1}{5}\tan 5x + C$$

ex: Evaluate.

i) $\int \frac{e^{2x} + 2e^x + 5}{e^x} dx$

$$\int (e^x + 2 + 5e^{-x}) dx$$

$$e^x + 2x + 5 \int e^{-x} dx$$

$$e^x + 2x + -5 \int e^u du = e^x + 2x - 5e^{-x} + C$$

ex: Evaluate.

k) $\int (\cos 2x - \sin 3x) dx$

$$\begin{aligned} & \int \cos 2x dx - \int \sin 3x dx \\ & \frac{1}{2} \cdot \sin(2x) - \frac{1}{3} \int \sin u \cdot \frac{du}{3} \quad \begin{array}{l} u = 3x \\ du = 3 dx \\ \frac{du}{3} = dx \end{array} \end{aligned}$$

$$\frac{1}{2} \sin(2x) + \frac{1}{3} \cos(3x) + C$$

ex: Evaluate.

$$\text{D) } \int \frac{x}{\sqrt[3]{1-2x^2}} dx = \int x(1-2x^2)^{-1/3} dx$$
$$u = 1-2x^2$$
$$du = -4x dx$$
$$\frac{du}{-4x} = dx$$
$$\left. \int x \cdot u^{-1/3} \frac{du}{-4x} \right)$$
$$- \frac{1}{4} \int u^{-1/3} du = -\frac{1}{4} \cdot \frac{u^{2/3}}{2/3}$$
$$= -\frac{3}{8} (1-2x^2)^{2/3} + C$$

ex: Evaluate.

m) $\int \cos^4(x) \sin(x) dx$

$$\begin{aligned} \int (\cos x)^4 \sin x dx &= \int u^4 \cdot \cancel{\sin x} \cdot \frac{du}{-\cancel{\sin x}} \\ u &= \cos x \\ du &= -\sin x dx \\ \frac{du}{-\sin x} &= dx \end{aligned}$$
$$\begin{aligned} - \int u^4 du &= -\frac{u^5}{5} + C \\ &= -\frac{(\cos x)^5}{5} + C \\ &= -\frac{\cos^5 x}{5} + C \end{aligned}$$

ex: Evaluate.

n) $\int \sin(x)\cos(x)dx$

ex: Evaluate.

$$q) \int x\sqrt{x+1} dx = \int (u-1)\sqrt{u} du$$

$$u = x + 1$$

$$u-1=x$$

$$du = dx$$

$$\int (u^{3/2} - u^{1/2}) du$$

$$\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C \leftarrow$$

$$\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$$

$$\int_0^1 x^3 (2x^4 + 1)^2 dx$$

#1

$$u = 2x^4 + 1$$

$$du = 8x^3 dx$$

$$\frac{1}{8} \int u^2 du$$

$$\frac{1}{8} \cdot \frac{u^3}{3} = \frac{1}{24} (2x^4 + 1)^3$$

$$\frac{1}{24} (27 - 1) = \frac{26}{24} = \boxed{\frac{13}{12}}$$

#2

$$u = 2x^4 + 1$$

$$du = 8x^3 dx$$

$$\frac{1}{8} \int_1^3 u^2 du$$

$$\frac{1}{8} \cdot \frac{u^3}{3}$$

$$\frac{1}{24} (27 - 1) = \frac{26}{24} = \boxed{\frac{13}{12}}$$

x	u
0	1
1	3

ex:

$$\int e^x \cos(e^x + 1) dx =$$

- (A) $\sin(e^x + 1) + C$
- (B) $e^x \sin(e^x + 1) + C$
- (C) $e^x \sin(e^x + x) + C$
- (D) $\frac{1}{2} \cos^2(e^x + 1) + C$

FR 1

A particle moves along the x -axis in such a way that its acceleration at time t for $t \geq 0$ is given by $a(t) = 4 \cos(2t)$. At time $t = 0$, the velocity of the particle is $v(0) = 1$ and its position is $x(0) = 0$.

- (a) Write an equation for the velocity $v(t)$ of the particle.
- (b) Write an equation for the position $x(t)$ of the particle.
- (c) For what values of t , $0 \leq t \leq \pi$, is the particle at rest?

Quiz topics:

1st FTC

2nd FTC

Average value ($1/(b-a)$)

U-substitution