4.4 The FUNdamental Theorem Of Calculus - cont.

 ex:

Let h be the function defined by $h(x)=\frac{1}{\sqrt{x^5+1}}$. If g is an antiderivative of h and $g(2)=3$, what is the value of $g(4)$?

(A) -0.020

(B) 0.152

(C) 3.031

(D) 3.152

4.4: FTC Part 2

- Accumulation Functions

The Definite Integral as a Number

Constant

$$\int_a^b f(x)\,dx$$

Constant

f is a function of x.

The Definite Integral as a Function of x

F is a function of x.

$$F(x) = \int_a^x f(t)\,dt$$

Constant

f is a function of t.

In general, an accumulation function comes in the form:

$$F(x) = \int_a^{g(x)} f(t)\,dt,$$

- The 2nd FUNdamental Theorem of Calculus

THEOREM 4.13 The Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_{a}^{g(x)} f(t)\,dt\right] =$$

*The 2nd FUNdamental Theorem of Calculus is used to DIFFERENTIATE an accumulation function.

Find F'(x).

$$F(x) = \int_{3}^{x} t^2 \, dt$$

Find F'(x)

$$F(x) = \int_{3}^{x^3} t^2 \, dt$$

Find F'(x)

$$F(x) = \int_{3}^{x^3} t^2 \, dt$$

Find F'(x)

$$F(x) = \int_{1}^{x} \sin^3 t \, dt$$

Find F'(x)

$$F(x) = \int_{1}^{x^2} \sqrt{t^2 + 5}\, dt$$

Find F'(x)

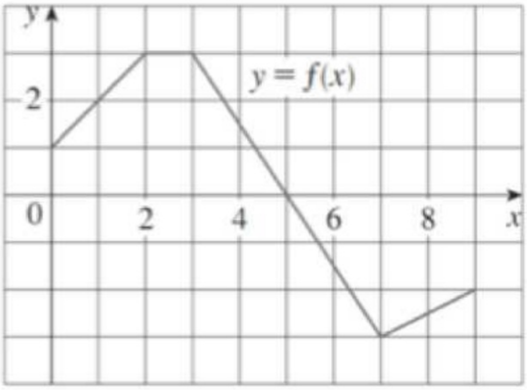
$$F(x) = \int_{x}^{2} \sqrt{t^2 + 5}\, dt$$

ex: If $f(x)=\int_{1}^{x}\frac{t^4+1}{t}dt$ find $f''(2)$.

4.4 Notes WKST

The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.

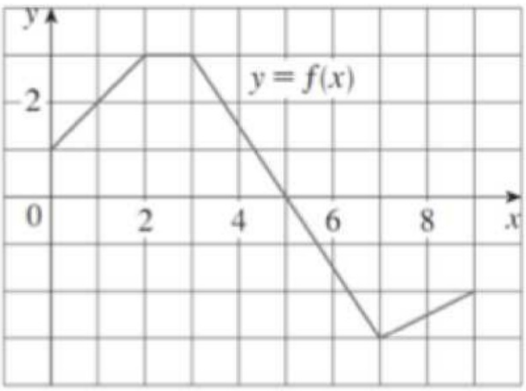
a) $g(0)$



*See printout.

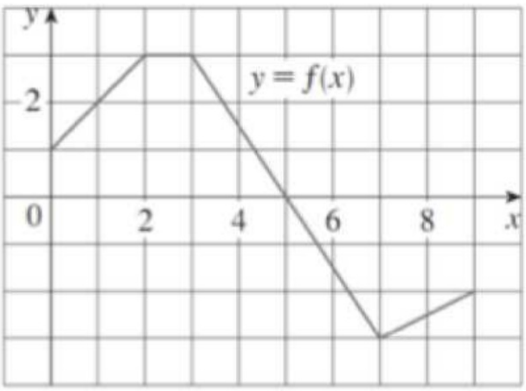
The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)dt$, evaluate the following or explain why they do not exist.

b) $g(5)$



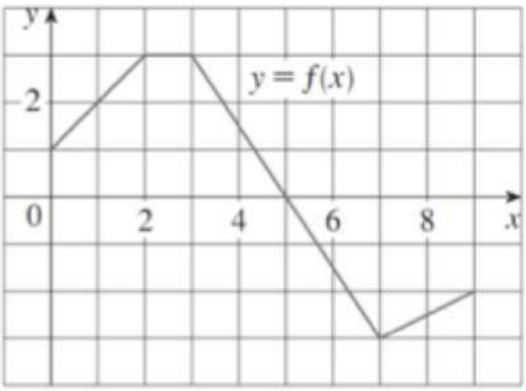
The graph of $f(x)$ is shown below. If $g(x)=\int_{2}^{x} f(t)\,dt$, evaluate the following or explain why they do not exist.

c) $g'(5)$



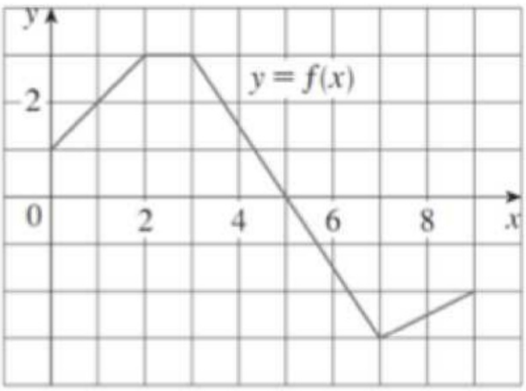
The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)dt$, evaluate the following or explain why they do not exist.

d) $g'(7)$



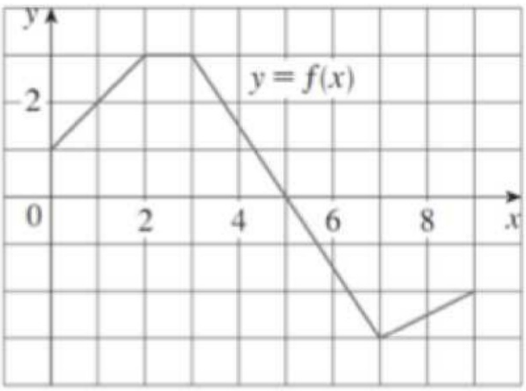
The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)dt$, evaluate the following or explain why they do not exist.

e) $g''(2)$

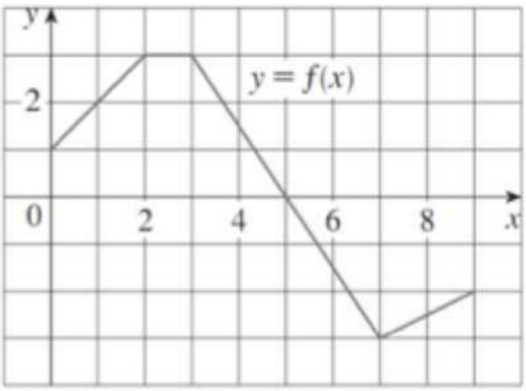


The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.

f) $g''(4)$

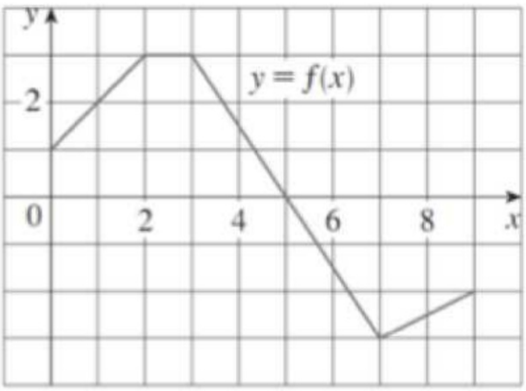


The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)dt$, evaluate the following or explain why they do not exist.



g) On what interval does $g(x)$ increase? Justify your answer

The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)dt$, evaluate the following or explain why they do not exist.



h) At what x-value(s) does $g(x)$ have a point of inflection? Justify your answer.

ex:

If $f(x) = \int_1^{x^3} \frac{1}{1+\ln t}\, dt$ for $x \geq 1$, then $f'(2) =$

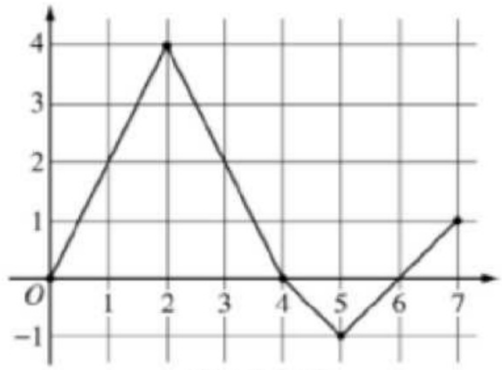
(A) $\frac{1}{1+\ln 2}$

(B) $\frac{12}{1+\ln 2}$

(C) $\frac{1}{1+\ln 8}$

(D) $\frac{12}{1+\ln 8}$

FR 16



Graph of f

Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

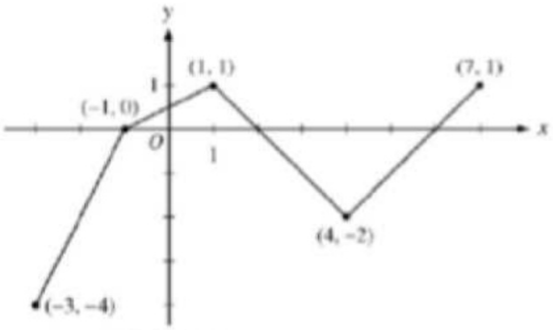
(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

FR 13

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.



Graph of g'

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

FR 20

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicates units of measure.

(d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?