4.4 The FUNdamental Theorem Of Calculus

THEOREM 4.11 The Fundamental Theorem of Calculus

If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx =$$

ex: Evaluate.

a) $\int_0^4 x^2 dx$

ex: Evaluate.

b) $\int_{1}^{4} \frac{3}{\sqrt{x}} dx$

ex: Evaluate.

c) $\int_{\pi/4}^{\pi/3} \left(\cot^2 x + 1\right) dx$

ex: Evaluate.

d) $\int_0^2 f(x)dx$ if $f(x) = \begin{cases} x^4, & x < 1 \\ x^5, & x \geq 1 \end{cases}$

ex: Evaluate.

e) $\int_0^3 |x-1|dx$

ex: Evaluate.

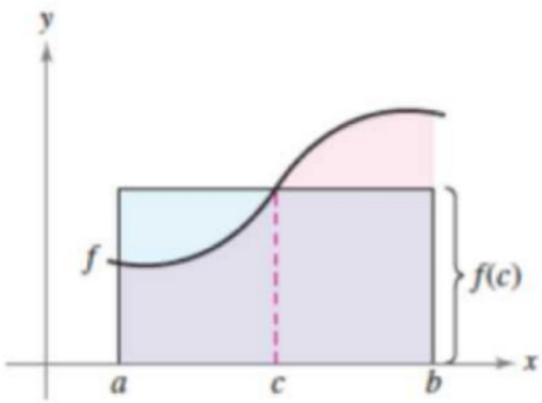
f) $\int_0^3 \sqrt{9-x^2}\,dx$

- Mean Value Theorem For Integrals

THEOREM 4.12 Mean Value Theorem for Integrals

If f is continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b - a).$$



The Mean Value Theorem for Integrals states that somewhere "between" the inscribed and circumscribed rectangles there is a rectangle whose area is precisely equal to the area of the region under the curve.

- Average Value

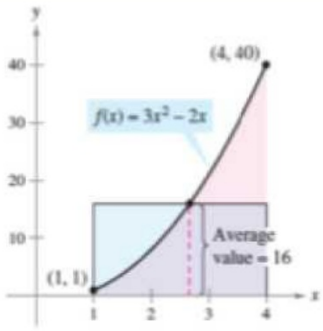
The value f(c) given in the Mean Value Theorem for Integrals is called the <u>average value</u> of f on the interval [a, b].

Definition of the Average Value of a Function on an Interval

If f is integrable on the closed interval $[a, b]$, then the **average value** of f on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

See Figure 4.36.



Average Value is not to be confused with Average Rate of Change!!!!

Average Value of f(x) on [a, b]:

Averate Rate of Change of f(x) on [a,b]:

ex: Find the average value of f(x) on the given interval

a) $f(x) = x^2, \quad [0,4]$

ex: Find the average value of f(x) on the given interval

b) $f(x)=\begin{cases} x^2-3, & x<3 \\ x+3, & x\geq 3 \end{cases}, \quad [0,5]$

Chocolate-Studded Dream Cookies

"The next part of this recipe will involve some calculus."

*See printout.