4.3 Definite Integration - Geometric Interpretation

- Definite Integral

$$\int_a^b f(x)dx$$

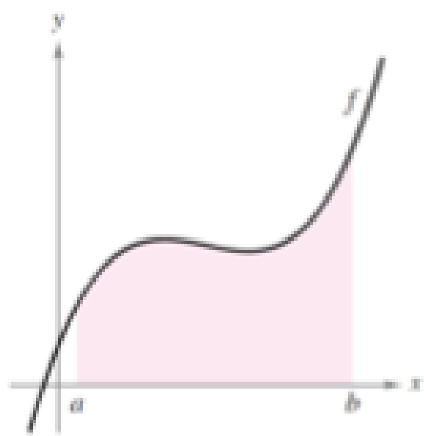
where

a - lower limit
b - upper limit

Definition - If f(x) is <u>continuous</u> and <u>nonnegative</u> on [a,b] then

$$\int_a^b f(x)dx$$

represents the area bounded by f(x), the x-axis, and the lines x=a and x=b.



You can use a definite integral to find the area of the region bounded by the graph of f, the x-axis, $x = a$, and $x = b$.

This area is often referred to as "the area under the curve."

ex: Evaluate.

a) $\int_{1}^{3} 4dx$

ex: Evaluate.

b) $\int_{0}^{3} (x+2)\,dx$

ex: Evaluate.

c) $\int_{-2}^{0} \sqrt{4 - x^2}\,dx$

ex: Evaluate.

d) $\int_{-1}^{3} |x-2| dx$

ex: Evaluate.

e) $\int_{-2}^{1} 2x\,dx$

ex: Evaluate.

f) $\int_{-5}^{5}\left(\sqrt{25-x^2}+4\right)dx$

ex: Evaluate.

g) $\int_{-2}^{2}\left(\sqrt{4-x^2}-2\right)dx$

ex: Evaluate.

h) $\int_{5}^{9}\left(\sqrt{4-(x-7)^{2}}\right)dx$

ex: Evaluate.

i) $\int_{-3}^{-1} -\sqrt{1-(x+2)^2}\,dx$

ex: Evaluate.

j) $\int_{0}^{5} f(x)dx$ if $f(x) = \begin{cases} -x-1, & x \le 2 \\ -3, & x > 2 \end{cases}$

Definite Integral Properties

If f(x) is continuous on [a,b] then…

1. $\int_{-a}^{a} f(x)dx =$

2. $\int_{b}^{a} f(x)dx =$

3. $\int_{a}^{b} kf(x)dx =$

Definite Integral Properties - cont.

4. $\int_a^b \left(f(x) \pm g(x)\right)dx =$

5. $\int_a^b f(x)dx =$

$a \le c \le b$

Definite Integral Properties - cont.

6. $\int_a^b \left(f(x)+c\right)dx =$

7. $\int_{a+c}^{b+c} f(x-c)dx =$

Definite Integral Properties - cont.

8. If f(x) is even, $\int_{-a}^{a} f(x)dx =$

9. If f(x) is odd, $\int_{-a}^{a} f(x)dx =$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

a) $\int_3^0 f(x)dx =$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

b) $\int_3^5 (f(x)+6)dx =$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

c) $\int_0^5 f(x)dx =$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

d) $\int_5^7 f(x-2)dx =$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

e) $\int_0^1 3f(x)dx =$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

f) $\int_1^3 f(x)dx =$

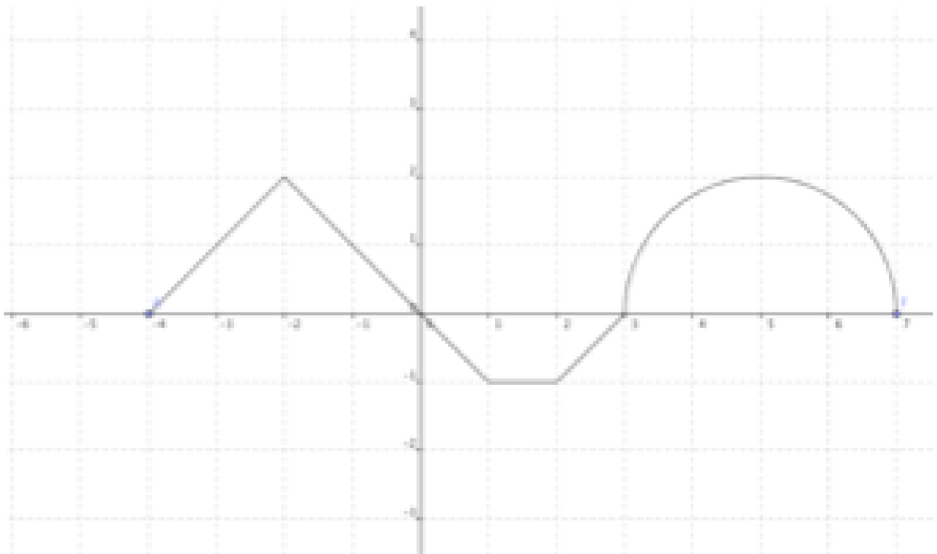
ex: Evaluate.

$$\int_{-13}^{13} \sin x dx =$$

ex: If f(x) is even and $\int_{0}^{20} f(x)dx = -7$ then

$$\int_{-20}^{20} f(x)dx =$$

ex: The graph below represents the function, $f(x)$.



Evaluate.

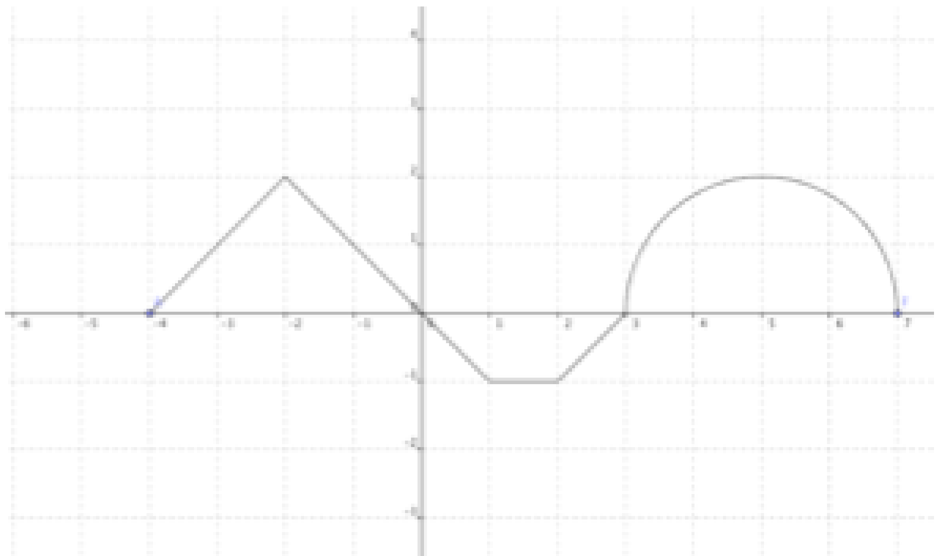
1. $\int_3^7 f(x)dx$

2. $\int_{-4}^0 f(x)dx$

3. $\int_{-4}^7 f(x)dx$

4. $\int_3^0 f(x)dx$

*See printout.



5. $\int_{-4}^{0}[f(x)+1]dx$

6. $\int_{3}^{7}-f(x)dx$

7. $\int_{-4}^{7}|f(x)|dx$

8. $\int_{3}^{-4}5f(x)dx$

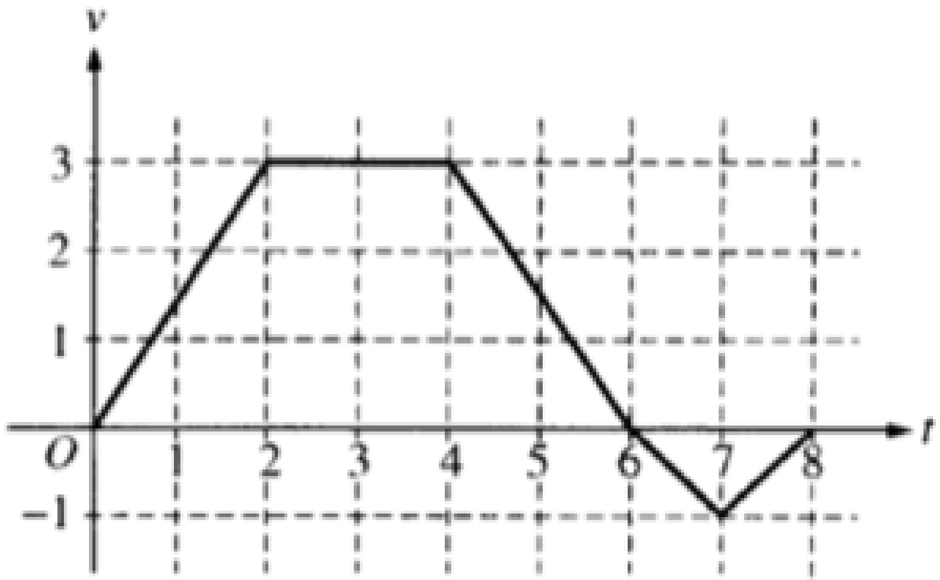
- Total Distance (by calculator)

ex: A particles moves on the x-axis so that its position at any time is given by: $x(t) = 4t^3 - 18t^2 + 15t - 1$

Find the total distance traveled by the particle from t=0 to t=3.

4.1-4.3 Extra Practice WKST

7. & 8.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

At what value of t does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

*See printout.

4.1-4.3 Extra Practice WKST

9.

If $\int_2^5 f(x)dx = 18$, then $\int_2^5 (f(x)+4)\,dx =$

(A) 20 (B) 22 (C) 23 (D) 25 (E) 30

4.1-4.3 Extra Practice WKST

10.

$$\int_{-4}^{4} (4-|x|)\,dx =$$

(A) 0 (B) 4 (C) 8 (D) 16 (E) 32

4.1-4.3 Extra Practice WKST

11.

If $\int_a^b f(x)dx = a + 2b$, then $\int_a^b (f(x)+3)dx =$

(A) $a+2b+3$ (B) $3b-3a$ (C) $4a-b$ (D) $5b-2a$ (E) $5b-3a$

4.1-4.3 Extra Practice WKST

12.

Given that $\int_4^9 \sqrt{x}dx = \frac{38}{3}$, using your knowledge of transformations, what is

(a) $\int_9^4 \sqrt{t}dt$

(b) $\int_4^9 \left(\sqrt{x}+3\right)dx$

(c) $\int_9^{14} \sqrt{x-5}dx$

(d) $\int_4^4 \sqrt{x}dx$

4.1-4.3 Extra Practice WKST

13. $f(x) = \begin{cases} x & \text{for } x < 2 \\ 3 & \text{for } x \geq 2 \end{cases}$

If f is the function defined above, then $\int_{-1}^{4} f(x)\,dx$ is

(A) $\frac{9}{2}$

(B) $\frac{15}{2}$

(C) $\frac{17}{2}$

(D) undefined

4.1-4.3 Extra Practice WKST

14.

A race car is traveling on a straight track at a velocity of 80 meters per second when the brakes are applied at time $t=0$ seconds. From time $t=0$ to the moment the race car stops, the acceleration of the race car is given by $a(t)=-6t^2-t$ meters per second per second. During this time period, how far does the race car travel?

(A) 188.229 m

(B) 198.766 m

(C) 260.042 m

(D) 267.089 m