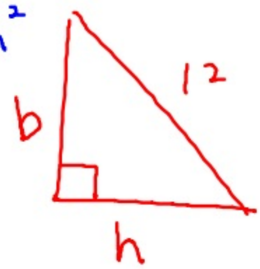
52.) primary: $A = \frac{1}{2}bh$

Secondary: $b^2 + h^2 = 12^2$

$b = \sqrt{144 - h^2}$



$A = \frac{1}{2}h\left(\sqrt{144-h^2}\right)$

33.) primary:

$$SA = 2\pi r h + 4\pi r^2$$

$$SA = 2\pi r\left(\frac{14-\frac{4}{3}\pi r^3}{\pi r^2}\right) + 4\pi r^2$$

$$SA = \frac{28}{r} - \frac{8}{3}\pi r^2 + 4\pi r^2$$

Secondary:

$$14 = \frac{4}{3}\pi r^3 + \pi r^2 h$$

$$\frac{14-\frac{4}{3}\pi r^3}{\pi r^2} = h$$

sphere + cylinder

$$\frac{4}{3}\pi r^3 + \pi r^2 h$$

53.) max volume

$$V = \frac{1}{3}\pi r^2 h$$

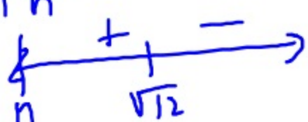
$$V = \frac{1}{3}\pi\left(36-h^2\right)h$$

$$V = 12\pi h - \frac{\pi}{3}h^3$$

$$V' = 12\pi - \pi h^2$$

$$12 = h^2$$

$$\sqrt{12} = h$$



secondary

$$h^2 + r^2 = 6^2$$

$$r^2 = 36 - h^2$$

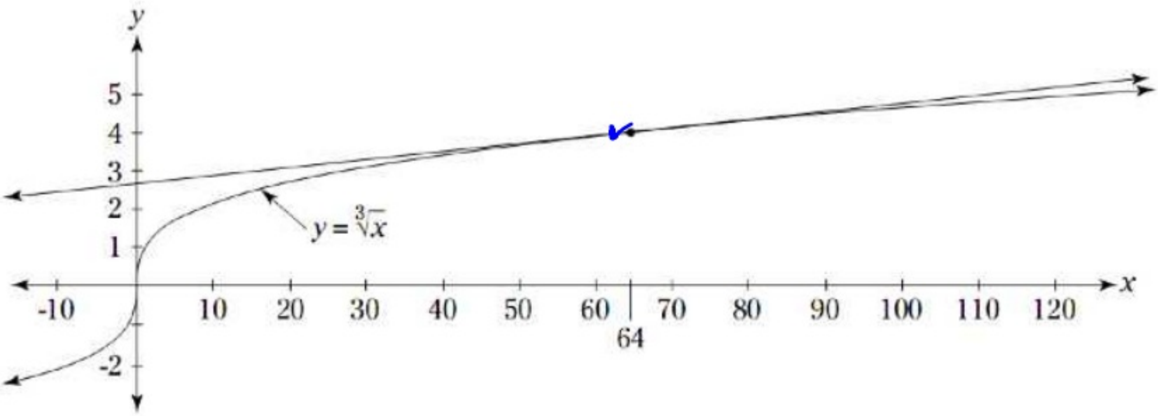
$$\boxed{\sqrt{12} = h \quad r = \sqrt{24}}$$

3.7—Linearization & Differentials

Linear approximation is a very easy thing to do, and once you master it, you can impress all of your friends by calculating things like $\sqrt[3]{70}$ in your head . . . . about 4.125! Impressed? I'll teach you how.

Recall that if a function $f(x)$ is differentiable at $x = c$, we say it is locally linear at $x = c$. This means that as we zoom in closer and closer and closer and closer around $x = c$, the graph of $f(x)$, regardless of how curvy it is, will begin to look more and more and more and more like the tangent line at $x = c$.

This means that we can use the equation of the tangent line of $f(x)$ at $x = c$ to approximate $f(c)$ for values close to $x = c$. Let's take a look at $\sqrt[3]{70}$ and the figure below.



Example 1:
Approximate $\sqrt[3]{70}$ by using a tangent line approximation centered at $x = 64$. Determine if this approximation is an over or under-approximation.

$f(x) = \sqrt[3]{x}$ $(64, 4)$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f'(64) = \frac{1}{3}(64)^{-2/3}$

$= \frac{1}{3} \cdot \frac{1}{16}$

$f'(64) = \frac{1}{48}$

Over approx because f''(70) < 0

To find a tangent line we need the slope at the point AND the point

$y - 4 = \frac{1}{48}(x - 64)$

$y - 4 = \frac{1}{48}(70 - 64)$

$y = \frac{1}{8} + 4 = 4\frac{1}{8} = 4.125$ Approx

Actual 4.121285

How to find linear approximations of $f(x)$ at $x = c$, the center to approximate $f(x)$ at $x = a$, a value near the center $x = c$.

1. Find the equation of the tangent line at the center $(c, f(c))$ in point-slope form.
2. Solve for y and rename it $L(x)$.
3. Plug in $x = a$ into $L(x)$ writing the notation VERY CAREFULLY as $f(a) \approx L(a) =$
4. If asked, determine if $L(a)$ is an over-approximation or an under approximation by examining the concavity of $f(x)$ at the center $x = c$.
   a. If $f''(c) < 0$, $f(x)$ is concave down at $x = c$ then $L(a)$ is an over-approximation
   b. If $f''(c) > 0$, $f(x)$ is concave up at $x = c$ and $L(a)$ is an under-approximation

Example 2:
Estimate the fourth root of 17. Determine if the linearization is and over- or under-approximation.

$(16, 2)$

$f(x) = \sqrt[4]{x}$

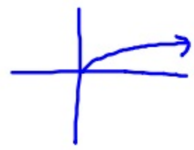
$f'(x) = \frac{1}{4}x^{-3/4}$

$f'(16) = \frac{1}{4}(16)^{-3/4}$

$= \frac{1}{32}$

$y - 2 = \frac{1}{32}(\underset{17}{x} - 16)$

$y = 2\frac{1}{32}$



$f''(16) < 0$ therefore the approx is over.

Example 3:
Approximate 3.01^5. Determine if the linearization is and over- or under-approximation.

$(3, 243)$

$f(x) = x^5$

$f'(x) = 5x^4$

$f'(3) = 405$

$f''(x) = 20x^3$

$f''(3) > 0$ (ccu)

under approx.

$y - 243 = 405(\underset{3.01}{x} - 3)$

$y = 405\left(\frac{1}{100}\right) + 243$

$y = \frac{405}{100} + 243$

$y = 4\frac{1}{20} + 243 = 247\frac{1}{20}$

Example 4:
Approximate $\ln\left(e^{10} + 5\right)$. Determine if the linearization is and over- or under-approximation.

$2 + \frac{1}{5}$

$2\frac{1}{5}$

$2 - \frac{1}{4}$

$1\frac{3}{4}$

$10 - \frac{1}{21}$

$9\frac{20}{21}$

$.01 = \frac{1}{100}$