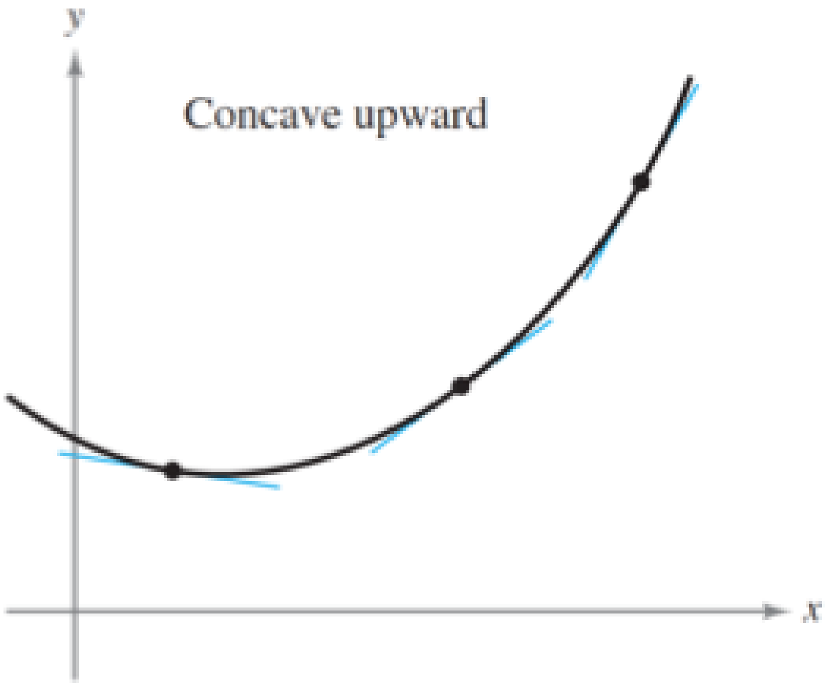
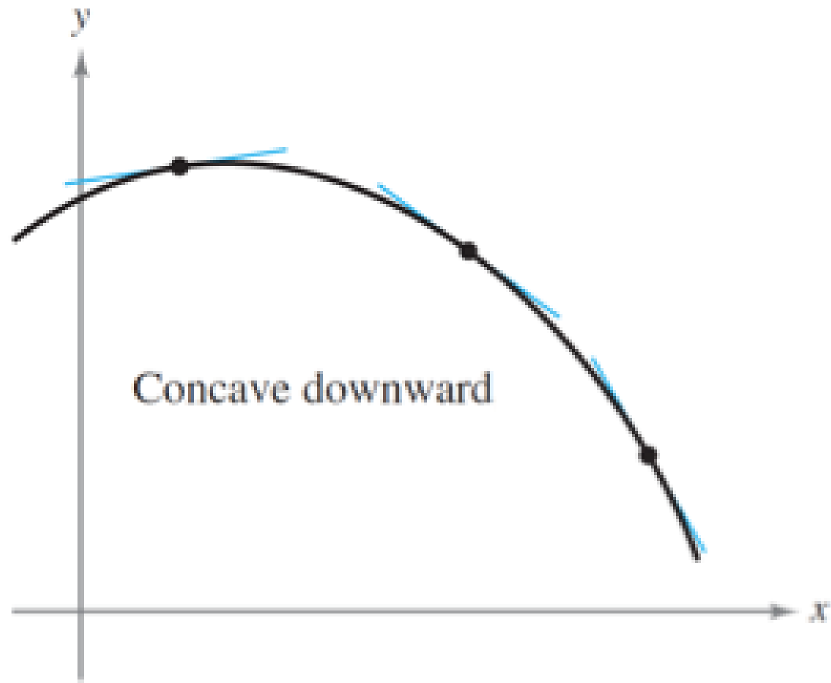
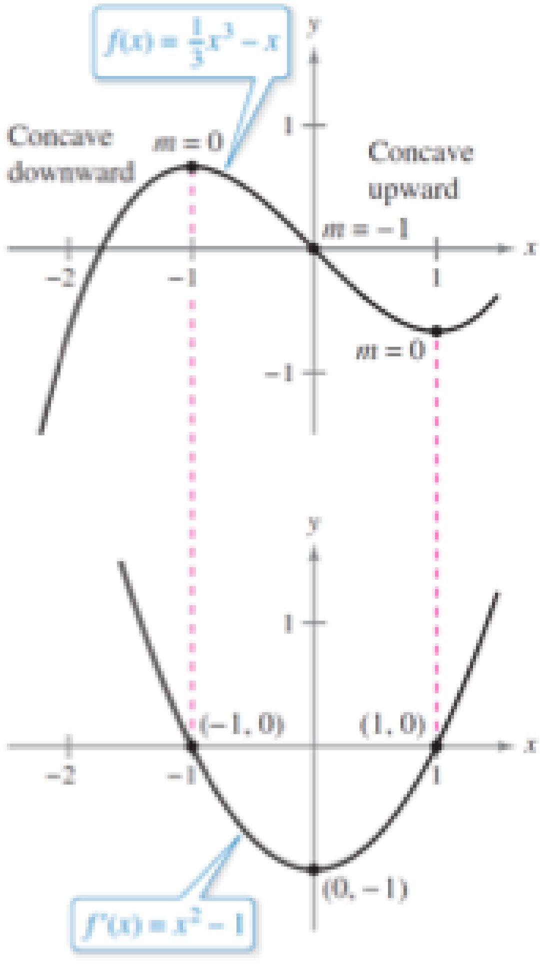
3.4 Concavity and the Second Derivative Test





Definition of Concavity

Let f be differentiable on an open interval I. The graph of f is **concave upward** on I when f' is increasing on the interval and **concave downward** on I when f' is decreasing on the interval.



f' is decreasing. f' is increasing.

f	CCU	CCD
f'		
f''		

ex: Determine the open intervals on which the graph of

$$f(x) = e^{-\frac{x^2}{2}}$$

is concave upward and concave downward? Justify your answer.

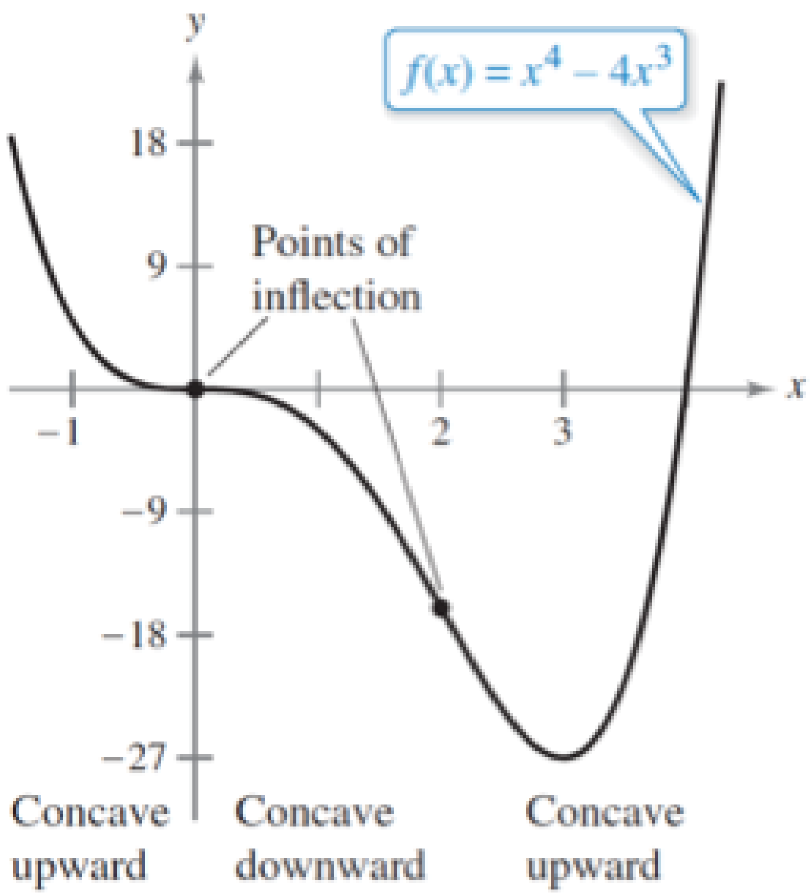
ex: Determine the open intervals on which the graph of

$$f(x) = \frac{x^2 + 1}{x^2 - 4}$$

is concave upward and concave downward? Justify your answer.

Definition of Point of Inflection

Let f be a function that is continuous on an open interval, and let c be a point in the interval. If the graph of f has a tangent line at this point $(c, f(c))$, then this point is a **point of inflection** of the graph of f when the concavity of f changes from upward to downward (or downward to upward) at the point.



POI REQUIREMENTS

1. f(c) is defined
2. f''(c) is 0 or undefined
3. f'' changes signs at x=c.

ex: Find all points of inflection on the graph of f(x), if possible.

$$f(x) = e^{-\frac{x^2}{2}}$$

ex: Find all points of inflection on the graph of f(x), if possible.

$$f(x)=\frac{x^2+1}{x^2-4}$$

f	CCU	CCD
f'		
f''		

When f has a point of inflection,

f' ____

f'' ____

3.4 WKST

The figures below show the graph of f'. For each of the functions find:

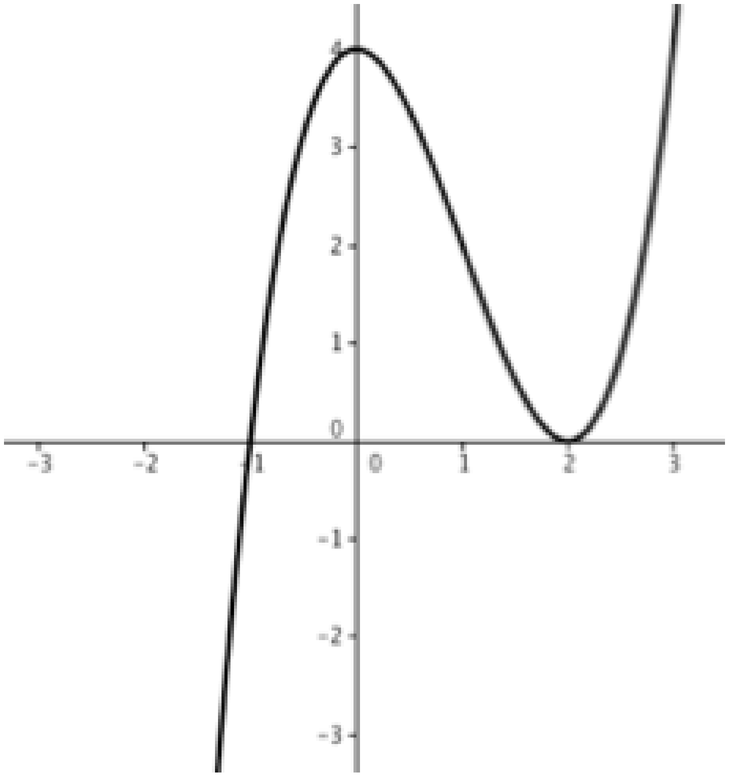
a) The intervals on which the graph of f is concave up.
b) The intervals on which the graph of f is concave down.
c) all x-values at which f has a point of inflection.

JUSTIFY all of your answers using f'.

*See printout.

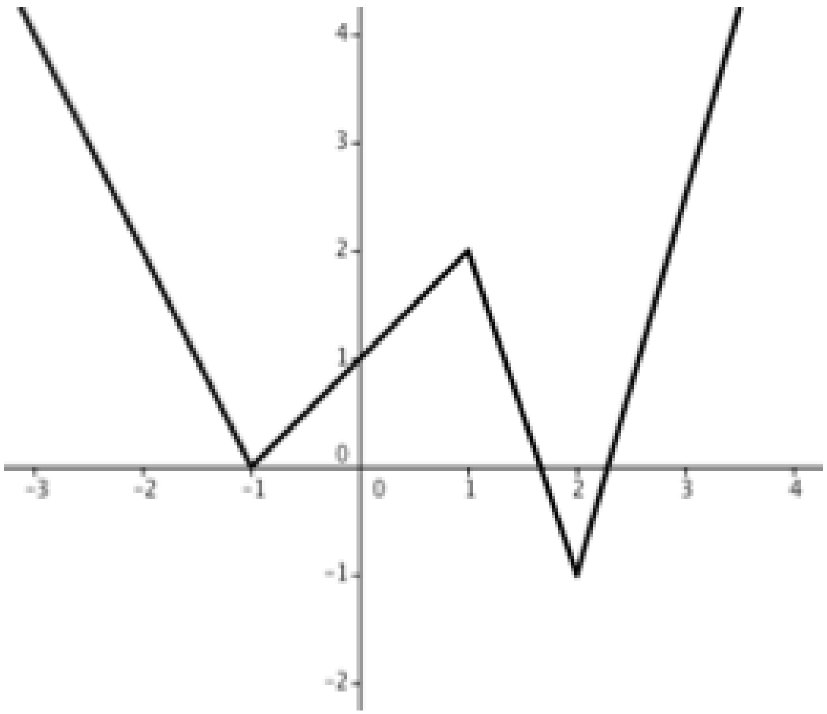
3.4 WKST

1a.



3.4 WKST

1b.

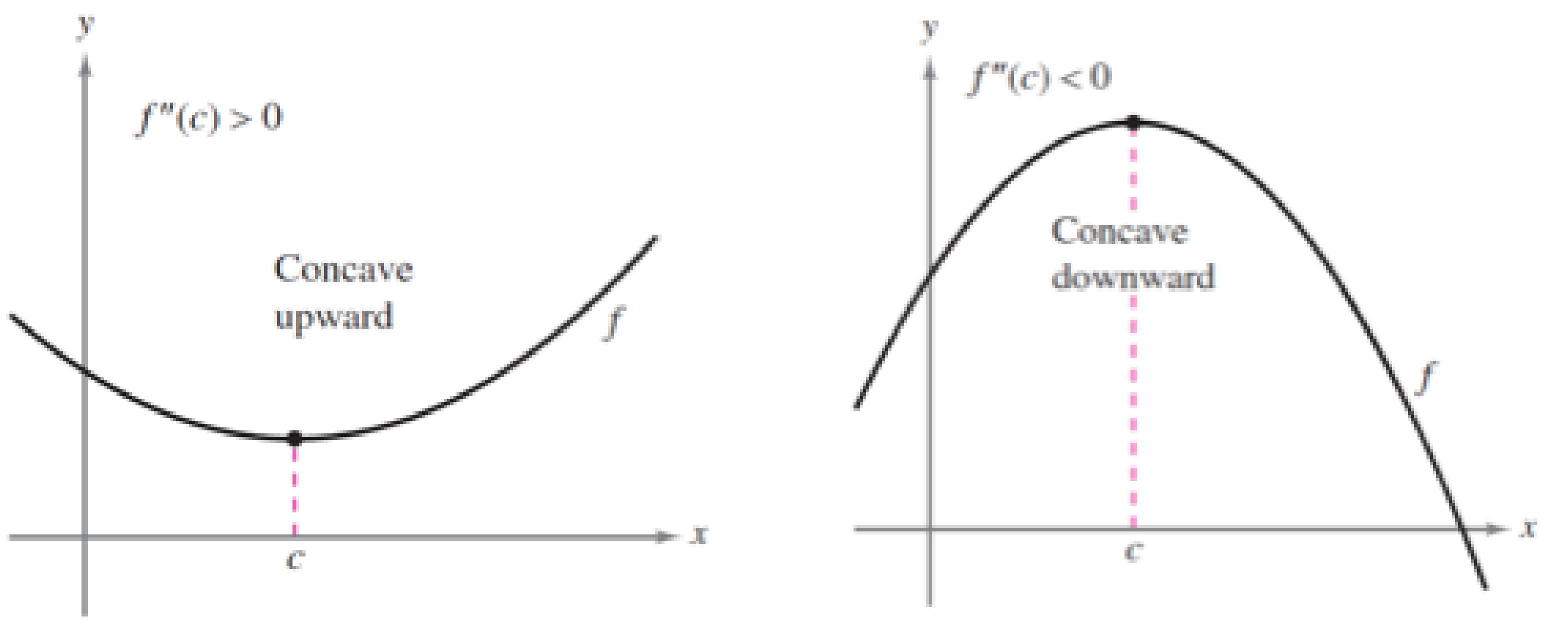


THEOREM 3.9 Second Derivative Test

Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then f has a relative minimum at $(c, f(c))$.
2. If $f''(c) < 0$, then f has a relative maximum at $(c, f(c))$.

If $f''(c) = 0$, then the test fails. That is, f may have a relative maximum, a relative minimum, or neither. In such cases, you can use the First Derivative Test.



ex: Use the 2nd Derivative Test to find all relative extrema of

$$f(x) = -3x^5 + 5x^3$$

ex: What can be concluded about f(x) at x=1 if

$$f(2)=16$$

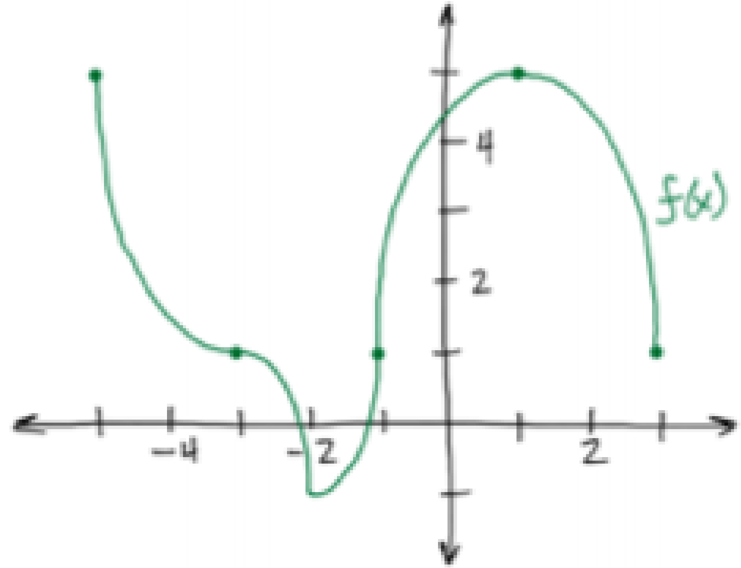
$$f'(2)=0$$

$$f''(2)=-300$$

3.4 WKST

2.

Let f be a continuous function on $[-5,3]$ with a vertical tangent line at $x=-1$, horizontal tangents at $x=-3$ and $x=1$ and a cusp at $x=-2$. The graph of f is given at right. Which of the following properties are satisfied?



I. $f''(x)<0$ on $(-2,1)$

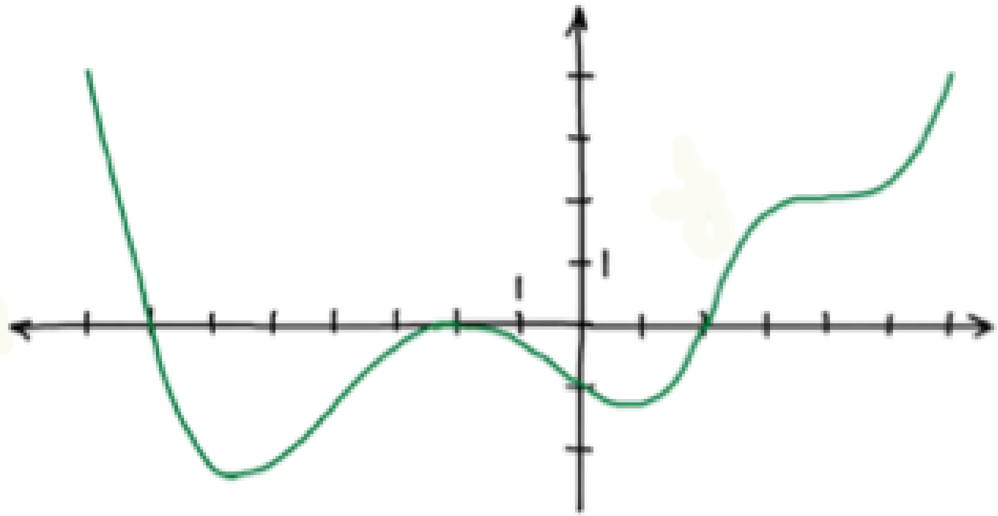
II. f has exactly 2 local extrema

III. f has exactly 4 critical points

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

3.4 WKST

3.



The graph of $f'(x)$

The figure above shows the graph of f', the derivative of function f, for $-8 < x < 6$. Of the following, which best describes the graph of f on the same interval?

(A) 1 local minimum, 1 local maximum, and 3 inflection points

(B) 1 local minimum, 1 local maximum, and 4 inflection points

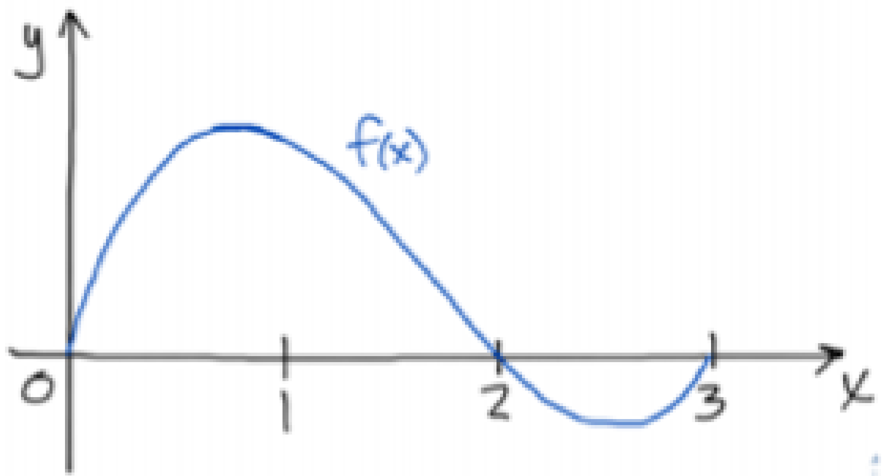
(C) 2 local minima, 1 local maximums, and 2 inflection points

(D) 2 local minima, 1 local maximum, and 4 inflection points

(E) 2 local minima, 2 local maxima, and 3 inflection points

3.4 WKST

4.



The graph of a differentiable function $f(x)$ is shown in the figure above and has an inflection point at $x = \frac{3}{2}$. Which of the following correctly orders $f(2)$, $f'(2)$, and $f''(2)$?

(A) $f(2) < f'(2) < f''(2)$

(B) $f'(2) < f(2) < f''(2)$

(C) $f'(2) < f''(2) < f(2)$

(D) $f''(2) < f(2) < f'(2)$

(E) $f''(2) < f'(2) < f(2)$

3.4 WKST

5.

If $f(0)=f'(0)=f''(0)=0$, which of the following **must** be true about the graph of f?

(A) There is a local max at the origin (B) There is a local min at the origin
(C) There is no local extremum at the origin (D) There is a point of inflection at the origin
(E) There is a horizontal tangent at the origin

3.4 WKST

6.

Let f be the function defined by $f(x) = 2x^3 - 3x^2 - 12x + 18$. On which of the following intervals is the graph of f both increasing and concave down?

(A) $(-\infty, -1)$ (B) $\left(-1, \frac{1}{2}\right)$ (C) $(-1, 2)$ (D) $\left(\frac{1}{2}, 2\right)$ (E) $(2, \infty)$

3.4 WKST

7.

If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x, which of the following could be a table of values for f?

(A)

x	$f(x)$
-1	4
0	3
1	1

(B)

x	$f(x)$
-1	4
0	4
1	4

(C)

x	$f(x)$
-1	4
0	5
1	6

(D)

x	$f(x)$
-1	4
0	5
1	7

(E)

x	$f(x)$
-1	4
0	6
1	7

3.4 WKST

8.

(Calculator Permitted) The derivative of the function f is given by $f'(x) = x^2 \sin\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

3.4 WKST

9.

The second derivative of a function g is given by $g''(x) = 2^{-x^2} + \cos x + x$. For $-5 < x < 5$, on what open intervals is the graph of g concave up?

(A) $-5 < x < -1.016$ only

(B) $-1.016 < x < 5$ only

(C) $0.463 < x < 2.100$ only

(D) $-5 < x < 0.463$ and $2.100 < x < 5$