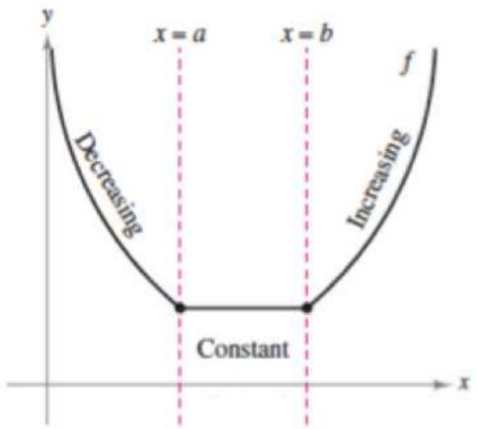
3.3 Increasing and Decreasing Functions and The First Derivative Test

Definitions of Increasing and Decreasing Functions

A function f is **increasing** on an interval when, for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

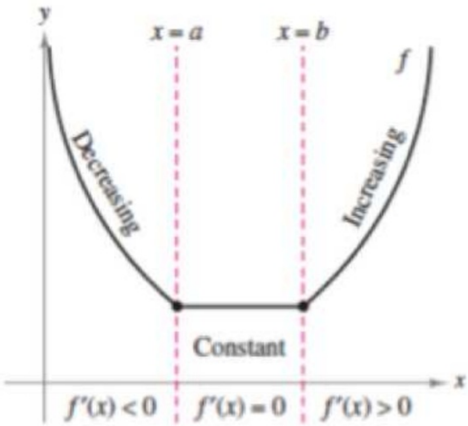
A function f is **decreasing** on an interval when, for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.



THEOREM 3.5 Test for Increasing and Decreasing Functions

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.



f	Inc	Dec
f'	+	−

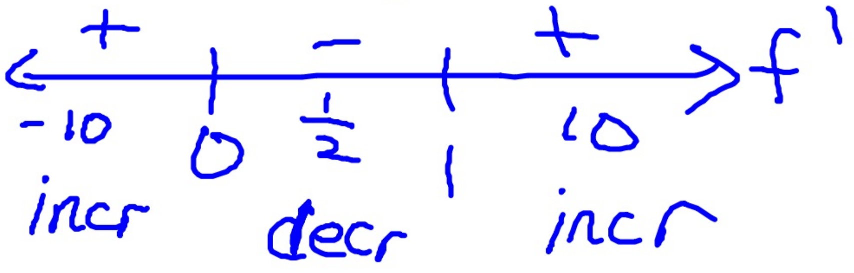
ex: On what interval(s) is f(x) increasing and decreasing? Justify your answer.

a) $f(x) = x^3 - \frac{3}{2}x^2$

$f'(x) = 3x^2 - 3x$

$0 = 3x(x-1)$

$x = 0, 1$



f is increasing on (-inf, 0) U (1, inf) because f'(x) > 0 on these intervals

f is decreasing on (0, 1) because f'(x) < 0 on this interval.

ex: On what interval(s) is f(x) increasing and decreasing? ~~Justify your answer.~~

b) $f(x) = xe^{-x}$

$D: (-\infty, \infty)$

$f'(x) = -xe^{-x} + e^{-x}$

$0 = -e^{-x}(x-1)$

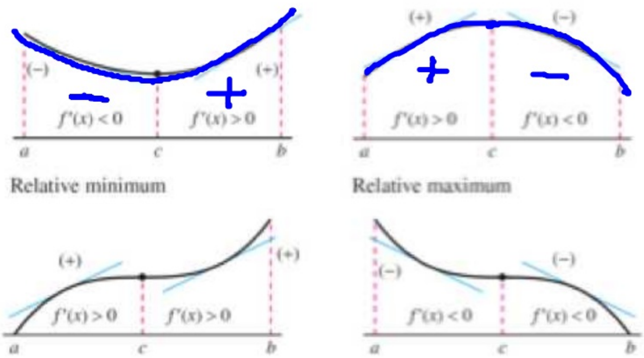
+		−	
0	1	10	f'
incr		decr	

Increasing on (-inf, 1); decreasing on (1, inf)

THEOREM 3.6 The First Derivative Test

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

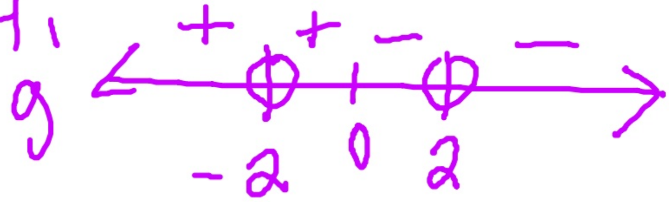
1. If $f'(x)$ changes from negative to positive at c, then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum.



Neither relative minimum nor relative maximum ex: Determine the value(s) of x in which f(x) has local extrema. Justify your answer.

a) $g(x) = \frac{x^2+1}{x^2-4}$

$g'(x) = \frac{-10x}{(x^2-4)^2}$



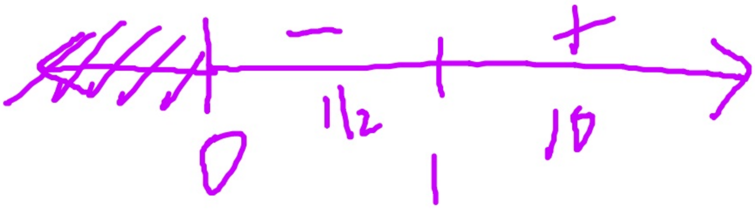
rel. max $x=0$

Relative maximum at x = 0 because f'(x) changes from positive to negative at x = 0.

ex: Determine the value(s) of x in which f(x) has local extrema. Justify your answer.

b) $f(x) = \frac{x^3}{3} - \ln 2x$

D: $(0, \infty)$



$f'(x) = \frac{x^3 - 1}{x}$

$x = 1$

Relative minimum x = 1 because f'(x) changes from negative to positive at this point.

ex: $f'(x)=(x-3)^4\cdot(x+1)^3$ odd (cross)

even (bounces)

a) At what x-values does f(x) have relative extrema? Justify your answer.

rel. min @ $x=-1$

Relative minimum at x = -1 because f'(x) changes from negative to positive at this point.