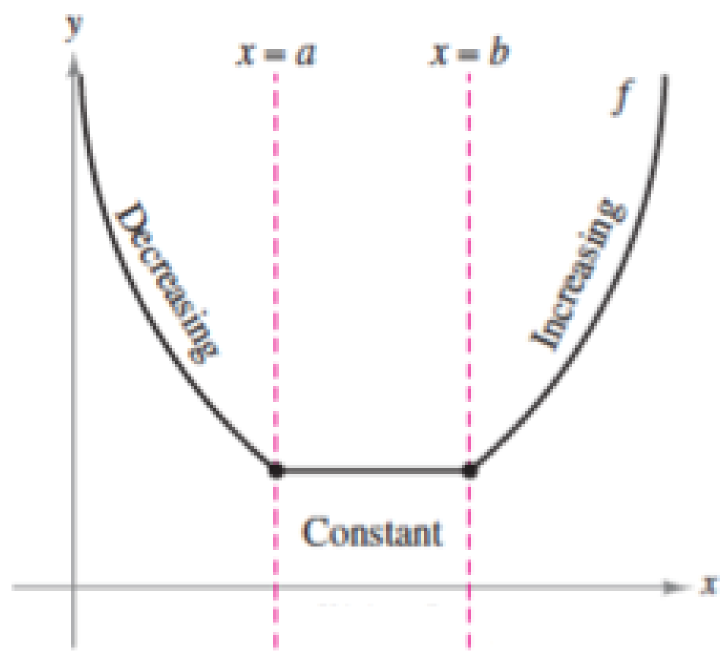
3.3 Increasing and Decreasing Functions and The First Derivative Test

Definitions of Increasing and Decreasing Functions

A function f is **increasing** on an interval when, for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

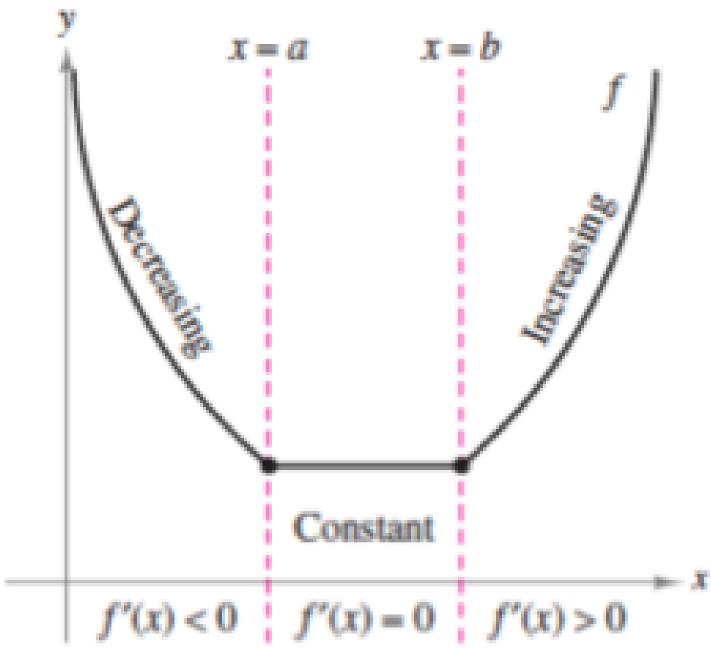
A function f is **decreasing** on an interval when, for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.



THEOREM 3.5 Test for Increasing and Decreasing Functions

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.



f	Inc	Dec
f'		

ex: On what interval(s) is f(x) increasing and decreasing? Justify your answer.

a) $f(x) = x^3 - \frac{3}{2}x^2$

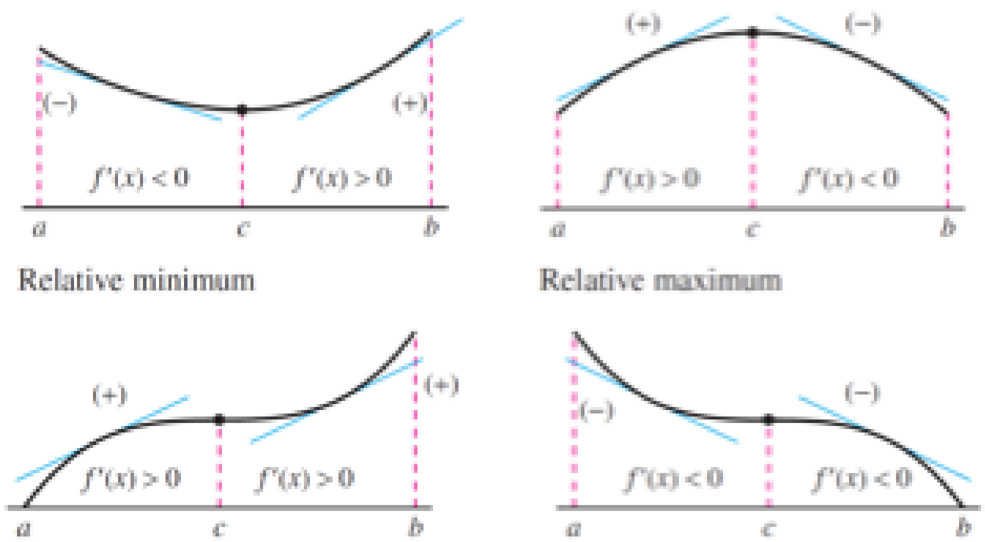
ex: On what interval(s) is f(x) increasing and decreasing? Justify your answer.

b) $f(x) = \sqrt{x}e^{-x}$

THEOREM 3.6 The First Derivative Test

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from negative to positive at c, then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum.



Neither relative minimum nor relative maximum ex: Determine the value(s) of x in which f(x) has local extrema. Justify your answer.

a) $f(x) = \dfrac{x^2}{x+1}$

ex: Determine the value(s) of x in which f(x) has local extrema. Justify your answer.

b) $f(x) = \dfrac{x^3}{3} - \ln 2x$

ex: $f(x)=\begin{cases} -x^3+1, & x\le 1 \\ x^2-4x, & x>1 \end{cases}$

a) At what x-values does f(x) have relative extrema? Justify your answer.

ex: $f(x) = \begin{cases} -x^3 + 1, & x \leq 1 \\ x^2 - 4x, & x > 1 \end{cases}$

a) On what intervals is f(x) increasing and decreasing? Justify your answer.

ex: $f(x)=(x-3)^{4/5}(x+1)^{1/5}$

a) At what x-values does f(x) have relative extrema? Justify your answer.

ex: $f(x) = (x-3)^{4/5}(x+1)^{1/5}$

a) On what intervals is f(x) increasing and decreasing? Justify your answer.

3.3 WKST

The figures below show the graph of f'. For each of the functions find:

a) all x-values of critical numbers of f.

b) intervals of increasing and decreasing on f.

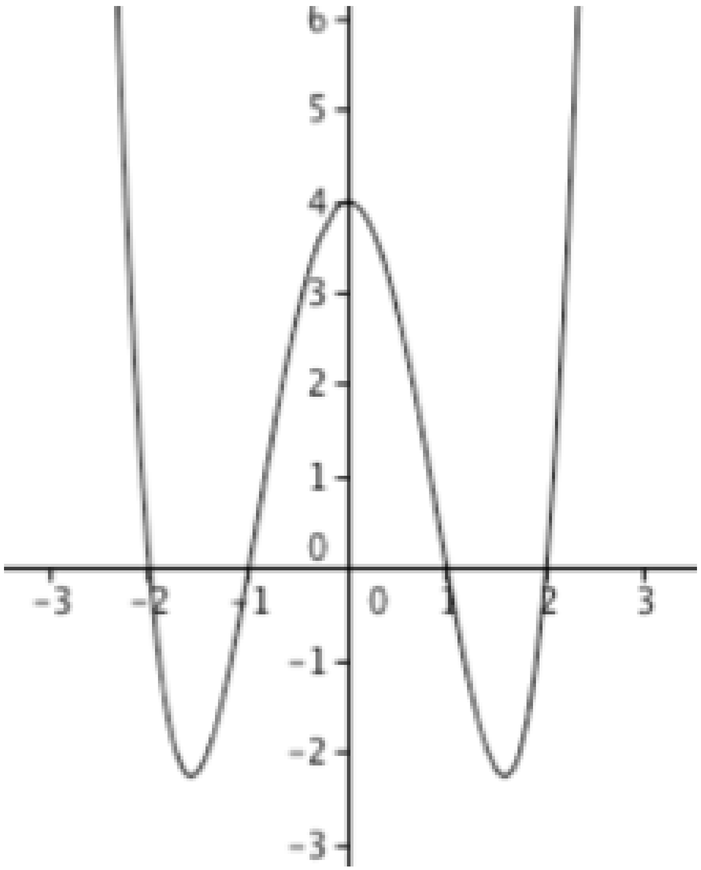
c) all x-values at which f has relative extrema.

JUSTIFY all of your answers using f'.

*See printout.

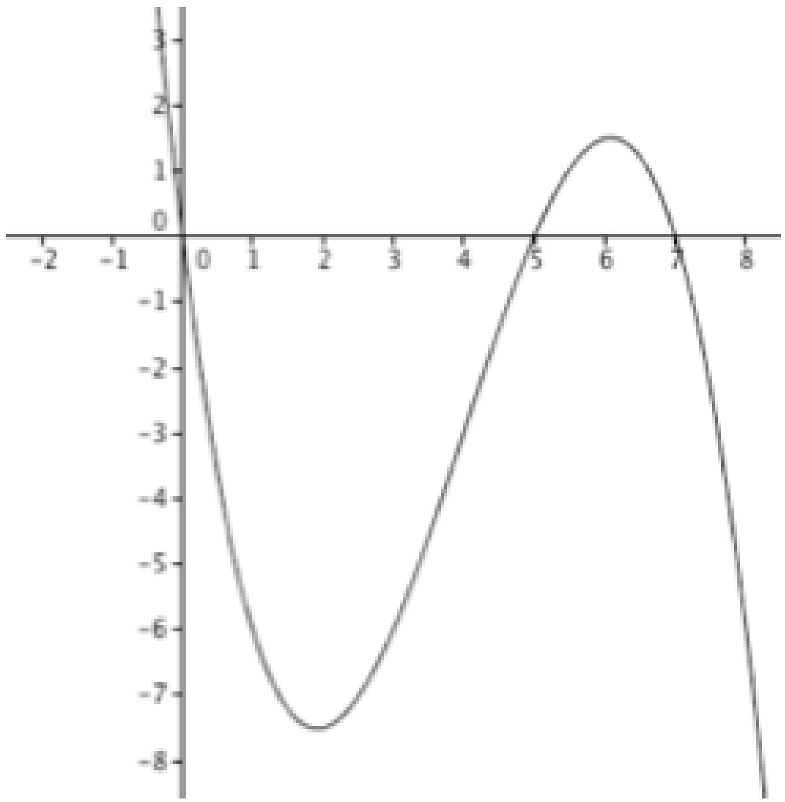
3.3 WKST

1.



3.3 WKST

2.



ex:

x	0	2	4	8
$f(x)$	3	4	9	13
$f'(x)$	0	1	1	2

The table above gives values of a differentiable function f and its derivative at selected values of x. If h is the function given by $h(x) = f(2x)$, which of the following statements must be true?

(I) h is increasing on $2 < x < 4$.

(II) There exists c, where $0 < c < 4$, such that $h(c) = 12$.

(III) There exists c, where $0 < c < 2$, such that $h'(c) = 3$.

(A) II only

(B) I and III only

(C) II and III only

(D) I, II, and III

ex:

The derivative of a function f is given by $f'(x) = e^{\sin x} - \cos x - 1$ for $0 < x < 9$. On what intervals is f decreasing?

(A) $0 < x < 0.633$ and $4.115 < x < 6.916$

(B) $0 < x < 1.947$ and $5.744 < x < 8.230$

(C) $0.633 < x < 4.115$ and $6.916 < x < 9$

(D) $1.947 < x < 5.744$ and $8.230 < x < 9$

 ex:

The derivative of the function f is given by $f'(x) = -\frac{x}{3} + \cos\left(x^2\right)$. At what values of x does f have a relative minimum on the interval $0 < x < 3$?

(A) 1.094 and 2.608

(B) 1.798

(C) 2.372

(D) 2.493