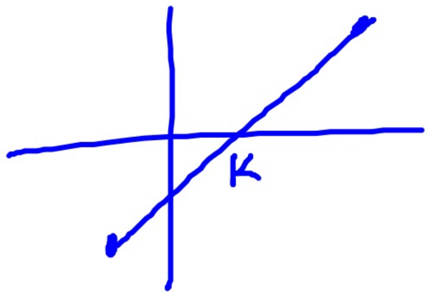
Tabular Data



Review: 4 Existence Theorems

1. IVT

Conditions: *Continuity on [a, b] and*
$f(a) \neq f(b)$ and $f(a) < k < f(b)$

Conclusion: *There must exist a value c in (a,b) such that $f(c) = k$.*

Review: 4 Existence Theorems

2. EVT

Conditions: *Continuity on [a, b]*

Conclusion: *There will be a maximum and minimum value on [a, b]*

Review: 4 Existence Theorems

3. MVT

Conditions: *Continuity on [a, b] and differentiability on (a, b)*

Conclusion: *There must exist a value c in (a, b) such that*

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Review: 4 Existence Theorems

4. Rolle's Theorem

Conditions: *continuity on [a, b]*

differentiability on (a, b)

$f(a) = f(b)$

Conclusion:

There must exist a value c in (a, b) such that $f'(c) = 0$.

Use the table below with selected values of the twice differentiable function k. Read each explanation and decide whether you would apply IVT, EVT, MVT, or Rolle's. (Since f is differentiable, f is also continuous).

x	1	2	3	4	5	6	7
f(x)	5	2	-4	-1	3	2	0

a. Since f(6) = 2 and f(7) = 0 and since 1 is between 2 and 0, it follows by IVT that f(c) = 1 for some c .

Use the table below with selected values of the twice differentiable function k. Read each explanation and decide whether you would apply IVT, EVT, MVT, or Rolle's. (Since f is differentiable, f is also continuous).

x	1	2	3	4	5	6	7
f(x)	5	2	-4	-1	3	2	0

b. Since $\frac{f(3)-f(2)}{3-2} = -6$, it follows by MVT that f'(c)=-6 for some c in the interval (2, 3).

[2,2) (3,-4)

$$\frac{-4-2}{3-2} = -6$$

Use the table below with selected values of the twice differentiable function k. Read each explanation and decide whether you would apply IVT, EVT, MVT, or Rolle's. (Since f is differentiable, f is also continuous).

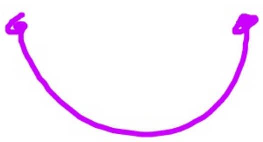
x	1	2	3	4	5	6	7
f(x)	5	2	-4	-1	3	2	0

c. There must be a minimum value for f at some r in [1, 7]. Hence the EVT applies.

Use the table below with selected values of the twice differentiable function k. Read each explanation and decide whether you would apply IVT, EVT, MVT, or Rolle's. (Since f is differentiable, f is also continuous).

x	1	2	3	4	5	6	7
f(x)	5	2	-4	-1	3	2	0

d. There must be some value a in (2, 6) for which $f'(a) = 0$ because $f(2) = f(6)$. Hence the Rollés applies.



ex: Consider the differentiable function v(t) with select values given in the table below.

t (min)	0	5	10	15	20	25	30
v(t) (meters/min)	7	9.2	9.5	7	4.5	2.4	2.4

a) Estimate a(7). Indicate units of measure.

$$a(7) \approx \frac{v(10)-v(5)}{10-5} = .06 \text{ m/min}^2$$

t (min)	0	5	10	15	20	25	30
v(t) (meters/min)	7	9.2	9.5	7	4.5	2.4	2.4

b) Estimate a(20). Indicate units of measure. m/min^2

$$\frac{v(25)-v(15)}{25-15} \quad \text{or} \quad \frac{v(20)-v(15)}{20-15} \quad \text{or} \quad \frac{v(25)-v(20)}{25-20}$$

$-.46$ $\qquad$ $-.5$ $\qquad$ $-.42$

t (min)	0	5	10	15	20	25	30
v(t) (meters/min)	7	9.2	9.5	7	4.5	2.4	2.4

c) What is the smallest number of instances in which $v(t)=8$ on $(0, 30)$? Justify your answer.

Twice. Since v(t) is differentiable it is also continous. IVT applies. $v(0) < 8 < v(5)$ and $v(15) < 8 < v(10)$. Therefore, there must be a value t on (0,5) such that $v(t) = 8$ but also on the interval (10, 15).

t (min)	0	5	10	15	20	25	30
v(t) (meters/min)	7	9.2	9.5	7	4.5	2.4	2.4

d) What is the smallest number of instances in which $a(t)=0$ on $(0, 30)$? Justify your answer.

Twice. Since differentiability implies continuity and $v(0) = v(15)$ and $v(25) = v(30)$, Rolle's Theorem applies. There must exist a value c in $(0, 15)$ and $(25, 30)$ such that $v'(c) = a(c) = 0$.

t (min)	0	5	10	15	20	25	30
v(t) (meters/min)	7	9.2	9.5	7	4.5	2.4	2.4

e) On the interval (0, 20) must there be a time when $a(t)=-1/8$? Justify your answer.

Since differentiability implies continuity, MVT applies.

$\dfrac{v(20) - v(0)}{20-0} = -1/8$, *there must exist a value c*

in (0, 20) such that $a(c) = v'(c) = -1/8$

Let y(t) represent the population of the town of Sugar Mill over a 10-year period where y is a differentiable function of t. The table below shows the population recorded every two years.

t(years)	0	2	4	6	8	10
y(people)	2500	2912	3360	3815	4330	4875

a. Approximate $y'(7)$ and explain the meaning of $y'(7)$ in terms of the population of Sugar Mill. Show the computations that lead to your conclusion. Include units of measure.

$$\frac{y(8)-y(6)}{8-6} = \frac{515}{2} = 257.5 \text{ people/year}$$

The rate of change of the population of Sugar Mill at t = 7 is estimated at 257.5 people/year.

Let y(t) represent the population of the town of Sugar Mill over a 10-year period where y is a differentiable function of t. The table below shows the population recorded every two years.

t(years)	0	2	4	6	8	10
y(people)	2500	2912	3360	3815	4330	4875

b. Find the average rate of change for y(t) on the interval (0, 10). Include units of measure.

$$\frac{y(10) - y(0)}{10 - 0} = 237.5 \text{ ppl/year}$$

Let y(t) represent the population of the town of Sugar Mill over a 10-year period where y is a differentiable function of t. The table below shows the population recorded every two years.

t(years)	0	2	4	6	8	10
y(people)	2500	2912	3360	3815	4330	4875

c. Explain why there must there be a time t in (0, 10) such that y(t) = 4000.

Since y is differentiable it is also continous and $y(0) = y(10)$ so IVT applies. Since $y(0) < 4000 < y(10)$ there must exist a value t in (0, 10) such that $y(t) = 4000$.

Existence Theorems - AP Style Questions

1.

Let $f(x) = x^3 - x - 1$. On the interval $[-1,2]$ where does the instantaneous rate of change f equal the average rate of change of f on that interval?

a) $-\frac{1}{2}$
b) -1, 1
c) 0
d) 1
e) $\frac{1}{2}$

*See printout.

2.

Which of the following functions below satisfy the conditions of the MVT?

I. $f(x)=\dfrac{1}{x+1}$, $[0,2]$ II. $f(x)=x^{1/3}$, $[0,1]$ III. $f(x)=|x|$, $[-1,1]$

a) I only
b) I and II only
c) I and III only
d) II only
e) II and III only

3.

If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

~~(C)~~ f has a minimum value on $a \le x \le b$.

~~(D)~~ f has a maximum value on $a \le x \le b$.

4.

Let g be a continuous function on the closed interval $[0,1]$. Let $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?

EVT abs. max

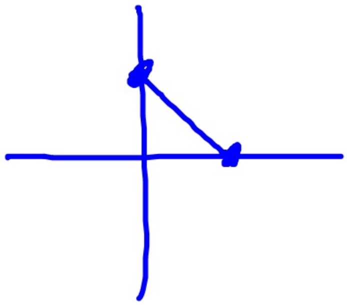
(A) There exists a number h in $[0,1]$ such that $g(h) \geq g(x)$ for all x in $[0,1]$.

(B) For all a and b in $[0,1]$, if $a=b$, then $g(a)=g(b)$.

(C) There exists a number h in $[0,1]$ such that $g(h)=\frac{1}{2}$.

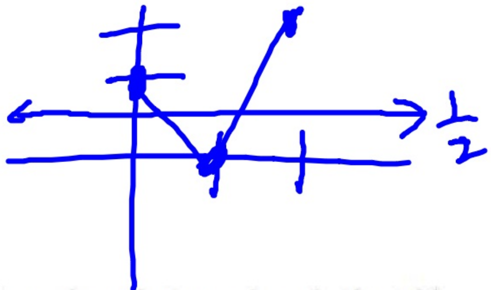
(D) There exists a number h in $[0,1]$ such that $g(h)=\frac{3}{2}$.

(E) For all h in the open interval $(0,1)$, $\lim_{x \to h} g(x)=g(h)$.



5.

x	0	1	2
$f(x)$	1	k	2



The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x)=\frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k=$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3