39.) $f(x) = \arctan x^2$ $[-2, 1]$

$$f'(x) = \frac{2x}{1+x^4}$$

$x = 0$

abs. max $(-2, \arctan 4)$

abs min $(0, 0)$

x	$f(x)$	
-2	$\arctan 4$	1.3
0	0	
1	$\frac{\pi}{4}$	.79

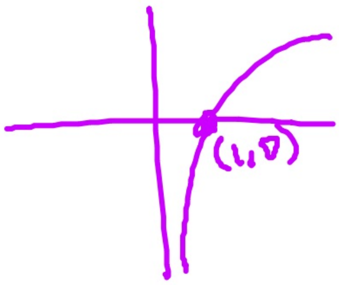
40.) $g(x) = \frac{\ln x}{x}$ $[1, 4]$

$$g'(x) = \frac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2}$$

$$g'(x) = \frac{1 - \ln x}{x^2}$$

$$0 = 1 - \ln x$$

$$e^{\ln x} = e^1 \quad x = e$$



x	$g(x)$
1	0
e	$1/e$ .36
4	$\frac{\ln 4}{4}$ .34

Abs. min $(1, 0)$

Abs max $(e, 1/e)$

7(a.)

$$f'(x) = \frac{4(1-3\ln x)}{x^4}$$

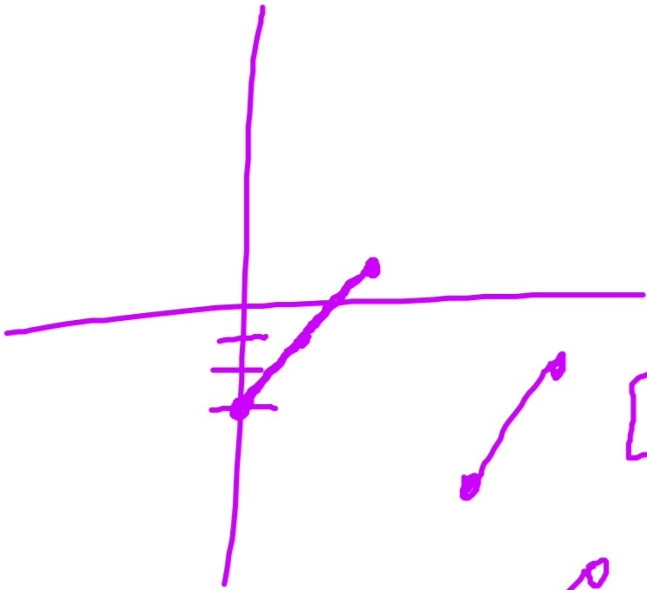
$$f'(e) = \frac{4(1-3\ln e)}{e^4} \quad \ln e^3$$

$$\left(e, \frac{4}{e^3}\right)$$

$$= \frac{4(1-3)}{e^4} = \frac{-8}{e^4}$$

$$y - \frac{4}{e^3} = \frac{-8}{e^4}(x-e)$$

4\.)



[0, 2]

[0, 2)

(0, 2]

(0, 2)

3.2 Rolle's Theorem and the Mean Value Theorem

ex: List the critical numbers of f(x).

$$f(x) = x^{4/5}(x-5)^2$$

ex: Find the maximum and miniumum values of f(x) on the indicated interval.

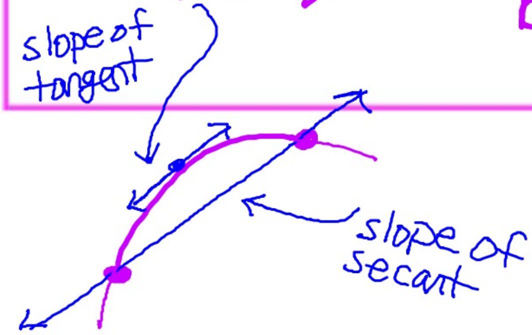
$$f(x) = \sin x + \cos x, \quad \left[0, \frac{\pi}{2}\right]$$

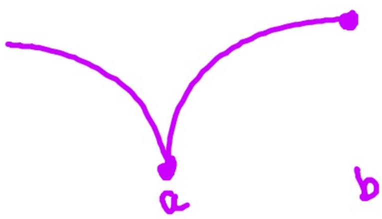
3.2

THEOREM 3.4 The Mean Value Theorem

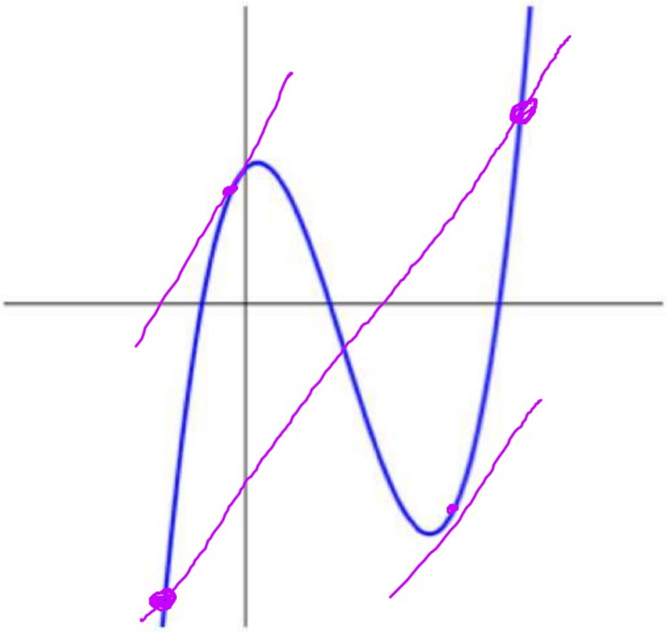
If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$





The Graphical Interpretation of the Mean Value Theorem.



ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

a) $f(x) = 5 - \frac{4}{x}, \quad [1,4]$

Cont. [1, 4] ✓

diff (1, 4) ✓

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$0 + 4x^{-2} = \frac{f(4) - f(1)}{4 - 1}$$

$$\frac{4}{x^2} = \frac{4 - 1}{3}$$

$$\frac{4}{x^2} = 1$$

$$x^2 = 4$$

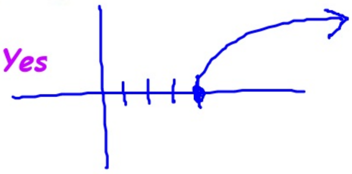
$$x = \pm 2$$

$$c = 2$$

ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

Continuous [4,8] Yes
Differentiable (4,8) Yes

b) $f(x) = \sqrt{x-4}, \quad [4,8]$



$$\frac{1}{2}(x-4)^{-1/2} = \frac{1}{2}$$

$$\frac{1}{\sqrt{x-4}} = 1$$

$$\sqrt{x-4} = 1$$

$$x-4 = 1$$

$$x = 5$$

ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

c) $f(x) = \frac{1}{x}, \quad [-1,1]$

MVT does not apply. The function is discontinuous at x = 0

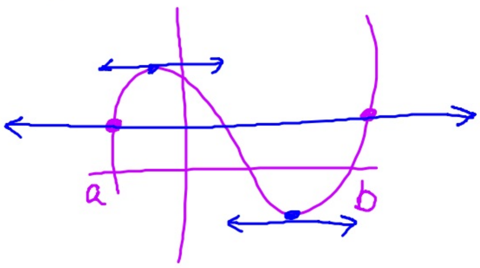
ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

d) $f(x) = |x|, \quad [-2,2]$

MVT does not apply. The function is not differentiable at x = 0

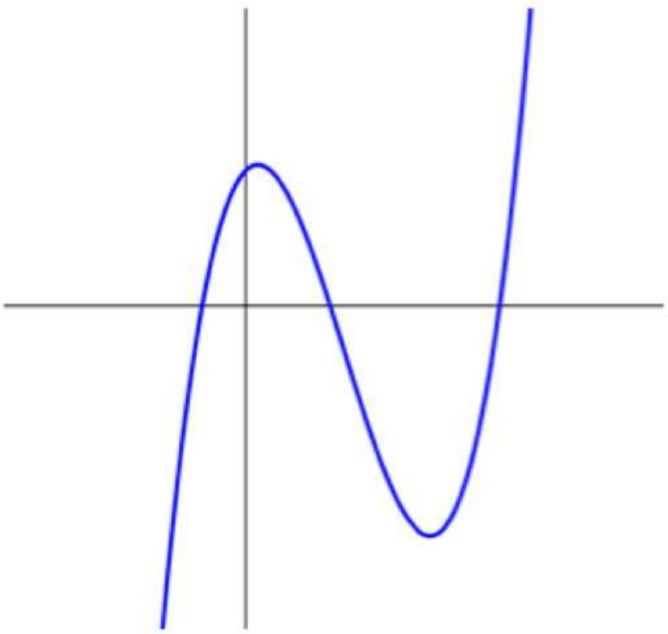
THEOREM 3.3 **Rolle's Theorem**

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b)$, then there is at least one number c in (a, b) such that $f'(c) = 0$



Rolle's Theorem is true because of Mean Value. The only difference is that the derivative is equal to 0.

The Graphical Interpretation of Rolle's Theorem.



ex: Determine the value(s) of c guaranteed by the conclusion of Rolle's Theorem on the given interval, if possible.

$=0$

a) $f(x) = \cos\frac{x}{3}, \quad [0, 6\pi]$

Cont on [0,6pi] yes
Diff on (0,6pi) yes
f(0) = f(6pi) yes (equals 1)

$f'(x) = -\sin\frac{x}{3} \cdot \frac{1}{3}$

$0 = -\sin\frac{x}{3} \cdot \frac{1}{3}$

$\sin\frac{x}{3} = 0$

$\frac{x}{3} = 0, \pi, 2\pi, 3\pi$

$x = \cancel{0}, 3\pi, \cancel{6\pi}, \cancel{9\pi}$

$c = 3\pi$

ex: Determine the value(s) of c guaranteed by the conclusion of Rolle's Theorem on the given interval, if possible.

cont ✓
diff ✓
$f(1) \neq f(20)$

b) $f(x) = x, \quad [1, 20]$

Rolle's Theorem does not apply because $f(1) \neq f(20)$

ex:

Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

$f(x) = x^3 - 3x^2$ $[0,3]$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = \cancel{0}, 2$

FR:

$f(x) = x^3 - 7x + 6$

Let f be the function given by $f(x) = x^3 - 7x + 6$.

(a) Find the zeros of f. $\frac{p}{q}$ zeros: $x = 1, 2, -3$

(b) Write an equation of the line tangent to the graph of f at $x = -1$. $(-1, 12)$ $m = -4$

$y - 12 = -4(x+1)$

(c) Find the number c that satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1, 3]$.

$3x^2 - 7 = 6$

$x^2 = \frac{13}{3}$

$x = \pm\sqrt{\frac{13}{3}}$

$\boxed{c = \sqrt{\frac{13}{3}}}$