3.1 Extrema on an Interval

Definition of Extrema

Let f be defined on an interval I containing c.

1. $f(c)$ is the **minimum of f on I** when $f(c) \leq f(x)$ for all x in I.
2. $f(c)$ is the **maximum of f on I** when $f(c) \geq f(x)$ for all x in I.

Types of Extrema

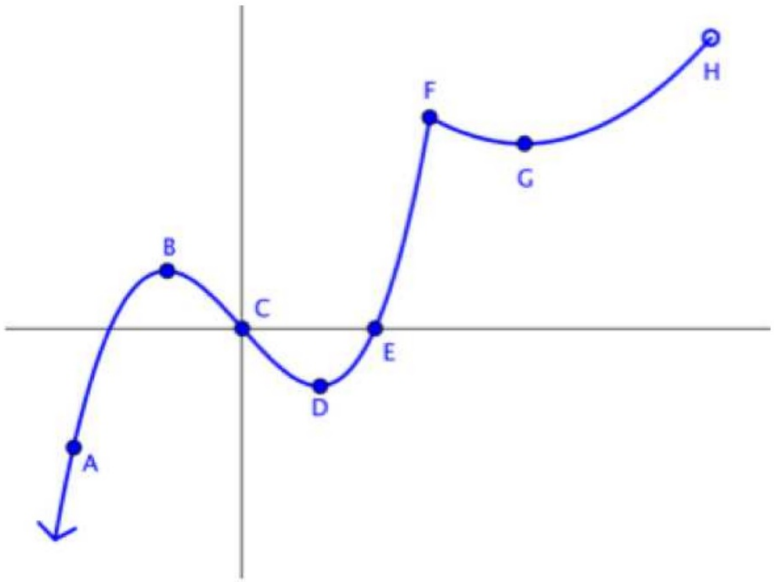
1. Absolute Extrema - the highest and lowest points on a curve (can occur ANYWHERE on a curve)

2. Relative Extrema - the highest and lowest points on a curve "in a neighborhood" (can occur ANYWHERE on an OPEN interval...no endpoints)
   (a.k.a. "Local Extrema)

When asked "where" does f(x) have extrema answer with an x-value.

When asked the extreme "value" of f(x) answer with a y-value.

ex:



At what point(s), if any, does f(x) have a(n)

a) Absolute Maximum

b)Absolute Minimum

c) Relative Maximum

d) Relative Minimum

ex: $y = \sin x$

a) Sketch on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

At what point(s), if any, does f(x) have a(n)

b) Absolute Maximum

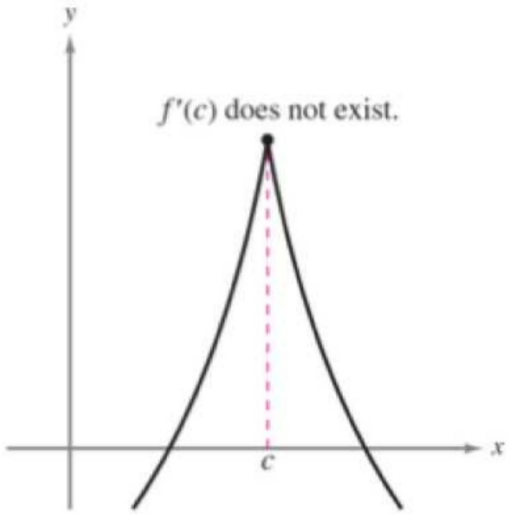
c)Absolute Minimum

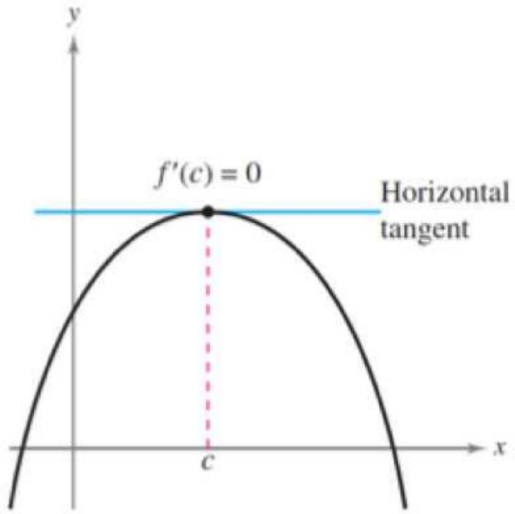
d) Relative Maximum

e) Relative Minimum

Definition of a Critical Number

Let f be defined at c. If $f'(c) = 0$ or if f is not differentiable at c, then c is a **critical number** of f.





ex: Find the critical numbers.

a) $f(x) = x^2 + 2x - 4$

ex: Find the critical numbers.

b) $f(x) = \sqrt[3]{x}$

ex: Find the critical numbers.

c) $f(x)=\frac{1}{x}$

ex: Find the critical numbers.

d) $f(x) = |x - 3|$

ex: Find the critical numbers.

e) $f(x) = 2x \ln x$

THEOREM 3.1 The Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

ex: Find the maximum and minimum values of f(x) on the indicated interval.

a) $f(x) = 3x^4 - 4x^3, \quad [-1,2]$

ex: Find the maximum and minimum values of f(x) on the indicated interval.

b) $f(x) = 2\sin x - \cos 2x, \quad [0, \pi]$

ex: What is the maximum acceleration on the interval [0,3] if the velocity is modeled by the equation

$$v(t) = t^3 - 3t^2 + 12t + 4$$

We need v''(t) to analyze what a(t) is doing

Find the maximum and minimum value of $y = 2x - 3x^{2/3}$ *on* $[-1,3]$

ex: Sketch a function with the given characteristics.

Relative minimum at $x = -1$

Critical number (but no extremum) at $x = 0$

Absolute maximum at $x = 2$

Absolute minimum at $x = 5$