2.7: Related Rates

Steps to solving a related rate problem

1) Draw a picture of the physical situation. Write down the given rates and values.

2) Write an equation that relates the quantities of intereset.

3) Take the derivative with respect to time of both sides of the equation.

4) Solve for the quanitity needed.

1) Find the derivative with respect to time.

$$\left(x^2 + y^2 - 2y - 4x = 0\right)\frac{d}{dt}$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} - 2\frac{dy}{dt} - 4\frac{dx}{dt} = 0$$

2) Air is being pumped into a spherical balloon at a rate of 5 cm^3/min. Find the rate of change of the radius when the diameter of the balloon is 20 cm.

$\frac{dV}{dt} = 5\,cm^3/min$

$\left.\frac{dr}{dt}\right|_{d=20cm} =$ ______

$r = 10cm$

$\frac{d}{dt}\left(V = \frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$

$5 = \frac{4}{3}\pi\, 3(10)^2 \frac{dr}{dt}$

$\frac{5}{400\pi}\ cm/min = \frac{dr}{dt}$

3)) A circle's area is increasing at a rate of 5 in²/min. At what rate is the radius increasing when the circumference is 40π in.

$\frac{dA}{dt} = 5 \, in^2/min$

$\left.\frac{dr}{dt}\right|_{C=40\pi \, in} = \underline{\qquad}$

$C = 2\pi r$

$40\pi = 2\pi r$

$20 = r$

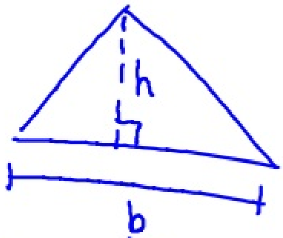
$\frac{d}{dt}\left(A = \pi r^2\right)$

$\frac{dA}{dt} = (2\pi r)\frac{dr}{dt}$

$5 = 40\pi \frac{dr}{dt}$

$\frac{5}{40\pi} \, in/min = \frac{dr}{dt}$

4) The altitude of a triangle is increasing at a rate of 1cm/min while the area of the triangle is increasing at a rate of 2cm^2/min. At what rate is the base of the triangle changing when the altitude is 10cm and the area is 100cm^2.



$\frac{dh}{dt} = 1\,cm/min.$

$\frac{dA}{dt} = 2cm^2/min$

$\left.\frac{db}{dt}\right|_{h=10cm,\,A=100cm^2} = \boxed{}$

$A = \frac{1}{2}bh$

$100 = \frac{1}{2}b(10)$

$20 = b$

$\frac{d}{dt}\left(A = \frac{1}{2}bh\right)$

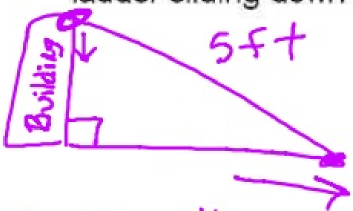
$\frac{dA}{dt} = \frac{1}{2}\left(b\cdot\frac{dh}{dt} + h\cdot\frac{db}{dt}\right)$

$2 = \frac{1}{2}\left(20\cdot 1 + 10\cdot\frac{db}{dt}\right)$

$4 = 20 + 10\frac{db}{dt}$

$-\frac{16}{10}\,cm/min = \frac{db}{dt}$

5) A 5 foot ladder is leaning against the side of a house when its base starts to slide away. By the time the base is 3 feet from the house, the base is moving at a rate of 1/4 ft/sec. How fast is the top of the ladder sliding down the wall at that moment?



$b = 3; \frac{db}{dt} = 1/4$ ft/sec

$\left.\frac{dh}{dt}\right|_{b=3ft} = \boxed{}$

$$b^2 + h^2 = c^2$$

$$\frac{d}{dt}\left(b^2 + h^2 = 5^2\right)$$

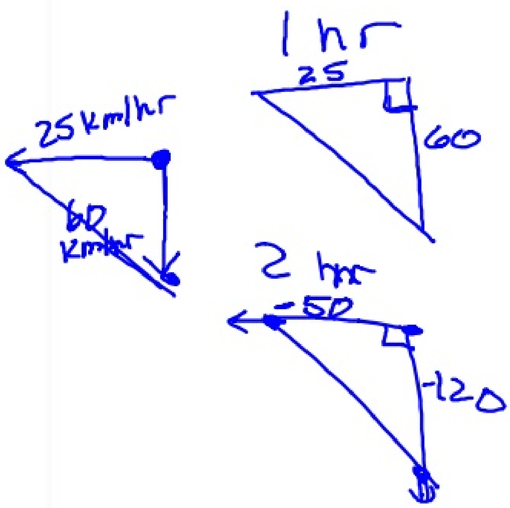
$$2b\frac{db}{dt} + 2h\frac{dh}{dt} = 0$$

$$b\frac{db}{dt} + h\frac{dh}{dt} = 0$$

$$(3)\left(\frac{1}{4}\right) + (4)\frac{dh}{dt} = 0$$

$$\frac{dh}{dt} = \frac{-3}{16} \text{ ft/sec}$$

6) Two cars start at the same point. One travels south at 60km/h and the other travels west at 25km/h. At what rate is the distance between them increasing two hours later?



$$\frac{dy}{dt} = -60 \text{ km/hr} \quad \frac{dx}{dt} = -25 \text{ km/hr}$$

$$\frac{d}{dt}\left(x^2 + y^2 = c^2\right)$$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}c\frac{dc}{dt}$$

$$(-50)(-25) + (-120)(-60) = 130\frac{dc}{dt}$$

$$65 \text{ km/hr} = \frac{dc}{dt}$$

51.) $x^2 - y^2 = 36$ $\quad \frac{dy}{dx} = \frac{x}{y}$

$$\frac{d^2y}{dx^2} = \frac{y - x \cdot \frac{dy}{dx}}{y^2}$$

$$= \frac{\left(y - x\left(\frac{x}{y}\right)\right) \cdot y}{(y^2)\, y} = \frac{y^2 - x^2}{y^3} = \frac{-(x^2 - y^2)}{y^3}$$

$$-\frac{36}{y^3}$$

25 b.) $\dfrac{16y^2 - x^2}{16} = \dfrac{16}{16} \longrightarrow y = \pm \dfrac{\sqrt{16+x^2}}{4}$

hyp. $\dfrac{y^2}{1} - \dfrac{x^2}{16} = 1$