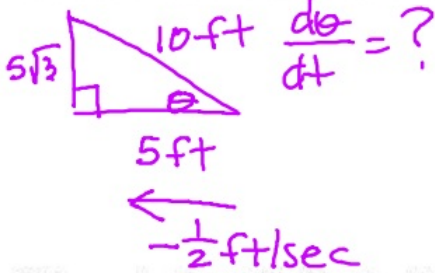


More Related Rates and Implicit Differentiation

1) A 10-foot plank is leaning against a wall. If at a certain instant the bottom of the plank is 5 feet from the wall and is being pushed toward the wall at a rate of $\frac{1}{2}$ ft/sec, how fast is the acute angle that the plank makes with the ground increasing?

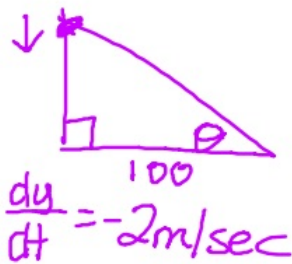


$$\cos \theta = \frac{x}{10} \quad \frac{d\theta}{dt} = -\frac{1}{20} = \frac{2}{10\sqrt{3}}$$

$$-\sin \theta \frac{d\theta}{dt} = \frac{1}{10} \frac{dx}{dt} = \frac{1}{10\sqrt{3}} \text{ rad/sec}$$

$$-\left(\frac{\sqrt{3}}{2}\right) \frac{d\theta}{dt} = \frac{1}{10} \left(-\frac{1}{2}\right)$$

3) You are looking at the New York ball drop on New Year's Eve at a distance of 100 m away from the base of the structure. If the ball drops at a constant rate of 2 m/s, what is the rate of change of the angle between you and the ball when the angle is $\pi/3$?

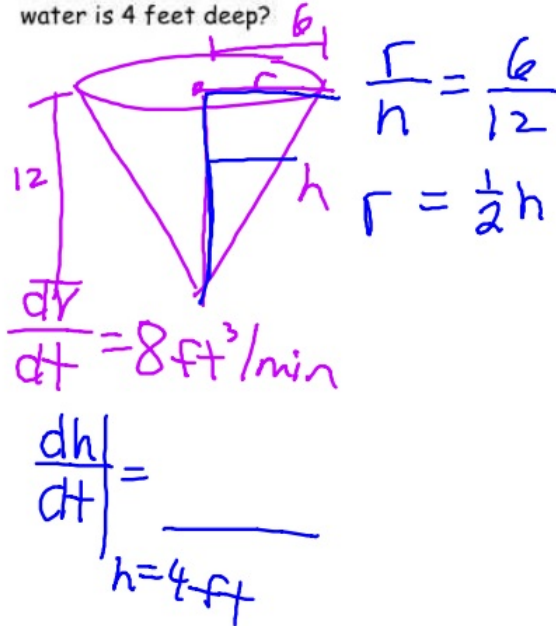


$$\tan \theta = \frac{y}{100} \quad \frac{d\theta}{dt} = -\frac{1}{50} \cdot \frac{1}{4}$$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{100} \frac{dy}{dt} = -\frac{1}{200} \text{ rad/sec}$$

$$4 \cdot \frac{d\theta}{dt} = \frac{1}{100} (-2)$$

3) Water is pouring into a conical tank at the rate of 8 cubic feet per minute. If the height of the tank is 12 feet and the radius of its circular opening is 6 feet, how fast is the water level rising when the water is 4 feet deep?



$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{\pi}{3} \left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{\pi}{12} h^3$$

$$\frac{dV}{dt} = \frac{3\pi}{12} h^2 \frac{dh}{dt}$$

$$8 = \frac{\pi}{4} (16) \frac{dh}{dt}$$

$$\frac{2}{\pi} \text{ ft}^3/\text{min} = \frac{dh}{dt}$$

19)

Consider the curve defined by $x^2 + xy + y^2 = 27$.

- (a) Write an expression for the slope of the curve at any point (x, y) .
- (b) Determine whether the lines tangent to the curve at the x -intercepts of the curve are parallel. Show the analysis that leads to your conclusion.
- (c) Find the points on the curve where the lines tangent to the curve are vertical.

20)

The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second.

- a.) At the time the radius of the sphere is 10 cm, what is the rate of increase of its volume?
- b.) At the time the volume of the sphere is 36π cubic cm, what is the rate of increase of the area of a cross section through the center of the sphere?
- c.) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?