43.) $\frac{d}{dx}\left(x+y-1=\ln(x^2+y^2)\right)$; $(1,0)$

$$1\cdot\cancel{\frac{dx}{dx}}+1\cdot\frac{dy}{dx}=\frac{1}{x^2+y^2}\cdot\left(2x\cancel{\frac{dx}{dx}}+2y\frac{dy}{dx}\right)$$

$$\frac{\left(1+\frac{dy}{dx}\right)}{1}=\frac{2x+2y\frac{dy}{dx}}{(x^2+y^2)}$$

$$\boxed{\frac{dy}{dx}=\frac{2x-x^2-y^2}{x^2+y^2-2y}}$$

$$x^2+y^2+\underbrace{x^2\frac{dy}{dx}}+\underbrace{y^2\frac{dy}{dx}}=2x+\underbrace{2y\frac{dy}{dx}}$$

$$x^2\frac{dy}{dx}+y^2\frac{dy}{dx}-2y\frac{dy}{dx}=2x-x^2-y^2$$

$$\boxed{y-0=-1(x-1)}$$

87.) $\frac{dy}{dx} = \frac{x+3}{4-4y}$

Vert.: $4-4y=0; y=1$

horiz: $x+3=0, x=-3$

$x^2+4y^2+6x-8y+9=0$

ellipse

$x^2+4+6x-8+9=0$

$x^2+6x+5=0$

$(x+5)(x+1)=0$ vert. tan.

$-5, -1$

$(-5,1)$ $(-1,1)$

63.) $\frac{d}{dx}\left(x^2 + x\underbrace{\arctan y}_{} = y - 1\right)$

$$2x + x \cdot \frac{1}{1+y^2} \cdot \frac{dy}{dx} + \arctan y \cdot 1 = 1 \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-2x - \arctan y}{\frac{x}{1+y^2} - 1} \qquad \left(\frac{-\pi}{4}, -1\right)$$

$$\left.\frac{dy}{dx}\right|_{\left(\frac{-\pi}{4}, -1\right)} = \frac{\frac{\pi}{2} - \left(\frac{-\pi}{4}\right)}{\frac{\frac{-\pi}{4}}{1+1} - 1}$$

Logarithmic Differentiation

When given a complicated equation it is often convenient to use logarithms as aids in differentiating nonlogarithmic functions. This process is called logarithmic differentiation.

Candidates for Logarithmic Differentiation:

- $y = \dfrac{(x-2)^2}{\sqrt{x^2+1}}$

- $y = x^{x-1}$

$$\ln(y) = \ln\left(\frac{(x-2)^2}{\sqrt{x^2+1}}\right)$$

$$\ln\frac{x}{y^2}$$

$$\ln x - 2\ln y$$

$$\left(\ln y = 2\ln(x-2) - \frac{1}{2}\ln(x^2+1)\right)\frac{d}{dx}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \left(2\cdot\frac{1}{x-2}\cdot 1 - \frac{1}{2}\cdot\frac{1}{x^2+1}\cdot 2x\right)\cdot y$$

$$\frac{dy}{dx} = \left(\frac{2}{x-2} - \frac{x}{x^2+1}\right)\cdot\frac{(x-2)^2}{\sqrt{x^2+1}}$$

ex: Differentiate.

b) $y = x^{x-1}$

$$\ln y = \ln x^{x-1}$$

$$\frac{d}{dx}\left(\ln y = (x-1)\ln x\right)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = (x-1)\cdot\frac{1}{x} + \ln x\cdot 1$$

$$\frac{dy}{dx} = \left(\frac{x-1}{x} + \ln x\right)\cdot x^{x-1}$$

2.7 Related Rates

GUIDELINES FOR SOLVING RELATED-RATE PROBLEMS

1. Identify all *given* quantities and quantities *to be determined*. Make a sketch and label the quantities.
2. Write an equation involving the variables whose rates of change either are given or are to be determined.
3. Using the Chain Rule, implicitly differentiate both sides of the equation *with respect to time t*.
4. *After* completing Step 3, substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.

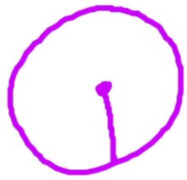
Think DREDS:

- Draw
- Rates
- Equation(s)
- Derive implicitly
- Substitute

$$\frac{d}{dt}\left(x^2 + y^3 = 4\right)$$

$$2x \cdot \frac{dx}{dt} + 3y^2 \cdot \frac{dy}{dt} = 0$$

#1 The radius of a circular oil slick expands at a rate of 2 m/min.



How fast is the area of the oil slick increasing when the radius is 25 m?

$\frac{dr}{dt} = 2\ m/min \qquad \frac{dA}{dt} = \_\_\_\_$ when $r = 25m$

$$\frac{d}{dt}\left(A = \pi r^2\right) \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2\pi(25)(2)$$

$$\frac{dA}{dt} = 100\pi\ m^2/min$$

#2 A sphere's volume is changing at a rate of 14 in^3/min. Determine the rate at which the radius is changing when the volume is 32pi cubic inches.

$\frac{dV}{dt} = 14\ in^3/min \quad \frac{dr}{dt} = \_\_\_$ when $V = 32\pi\ in^3$

$\frac{d}{dt}\left(V = \frac{4}{3}\pi r^3\right)$

$32\cancel{\pi} = \frac{4}{3}\cancel{\pi} r^3$

$\sqrt[3]{24} = \sqrt[3]{r^3}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\sqrt[3]{24} = r$

$14 = 4\pi\left(\sqrt[3]{24}\right)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{14}{4\pi \cdot 24^{2/3}}$ in/min

#3

All edges of a cube are expanding at a rate of 6 cm/sec.

$V = x^3$

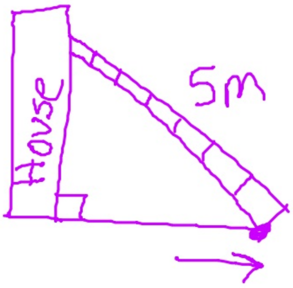
How fast is the volume changing when each edge is 2cm?

$\frac{dx}{dt} = 6\text{cm/sec}$ $\quad \frac{dV}{dt} = \_\_\_\_$ when $x = 2\text{cm}$

$$\frac{d}{dt}\left(V = x^3\right)$$

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt} = 3(2)^2(6) = 72\,\text{cm}^3/\text{sec}$$

#4 A 5 meter long ladder is leaning against the side of a house. The foot of the ladder is pulled away from the house at the rate of 0.4 m/sec. How fast is the top of the ladder descending when the foot of the ladder is 3 m from the house?



$\frac{dx}{dt} = 0.4$ m/sec $\quad \frac{dy}{dt} = $ ___ when $x = 3$m

Pythagoream Theorem

$$\frac{d}{dt}\left(x^2 + y^2 = 5^2\right) \Rightarrow \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$$

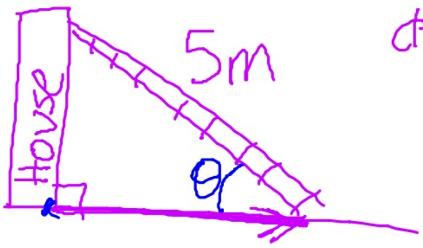
$$(3)(0.4) + (4)\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-3\left(\frac{2}{5}\right)}{4}$$

$$= \frac{-3}{10} \text{ m/sec}$$

#5 A 5 meter long ladder is leaning against the side of a house. The foot of the ladder is pulled away from the house at the rate of 0.4 m/sec. Find the rate at which the angle between the ladder and the ground is changing when the base of the ladder is 4 meters from the wall.

$\frac{dx}{dt} = 0.4 \text{ m/sec}$ $\quad \frac{d\theta}{dt} = \_\_\_$ when $x = 4\text{m}$



$$\cos\theta = \frac{x}{c}$$

$$\frac{d}{dt}\left(\cos\theta = \frac{x}{5}\right)$$

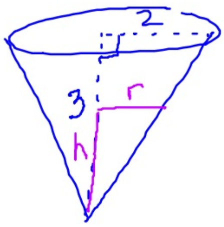
$$-\sin\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$$

$$-\frac{3}{5} \cdot \frac{d\theta}{dt} = \frac{1}{5} \cdot \frac{2}{5}$$

$$\frac{d\theta}{dt} = \frac{-2}{15} \text{ rad/sec}$$

#6 Two cars start moving from the same point. One travels south at 60 mi/h and the other travels east at 25 mi/hr. At what rate is the distance between the cars increasing two hours later?

#7 A conical tank has height 3 m and radius 2 m at the top.
Water flows in at a rate of 2 cubic meters per minute.
How fast is the water level rising when the height is 2m?



$\frac{dV}{dt} = 2\ m^3/min$ $\quad \frac{dh}{dt} = \_\_\_$ when $h = 2m$

Similar triangles

$$\frac{2}{3} = \frac{r}{h}$$

$$r = \frac{2h}{3}$$

$$V = \frac{1}{3}\pi r^2 h$$

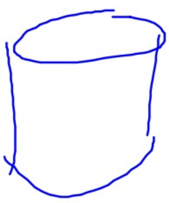
$$V = \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h = \frac{\pi}{3}\cdot\frac{4h^3}{9} = \frac{4\pi}{27}h^3$$

$$\frac{dV}{dt} = \frac{12\pi}{27}h^2\frac{dh}{dt}$$

$$2 = \frac{4\pi}{9}(4)\frac{dh}{dt}$$

$$\frac{9}{8\pi}\ m/min = \frac{dh}{dt}$$

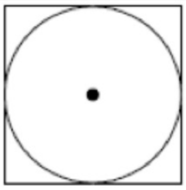
#8 . The radius of a right circular cylinder is increasing at a rate of 2 *in/min* and the height is decreasing at a rate of 3 *in/min.* At what rate is the volume changing when the radius is 8 *in* and the height is 12 *in*? Is the volume increasing or decreasing?



$\frac{dr}{dt} = 2 \text{ in/min}$ $\quad \frac{dh}{dt} = -3 \text{ in/min}$ $\quad \frac{dV}{dt} = \_\_$ when $r = 8$ in, $h = 12$ in

$$\frac{d}{dt}\left(V = \pi r^2 h\right)$$

$$\frac{dV}{dt} = \pi\left(r^2 \cdot 1 \cdot \frac{dh}{dt} + h \cdot 2r \frac{dr}{dt}\right)$$



A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle **expands**, the square **expands** to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$)

(a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

(b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.