4.) $f(x) = x^3 - 3x^2 + 8x + 5$

$5 = x^3 - 3x^2 + 8x + 5$

$0 = x$

$f'(x) = 3x^2 - 6x + 8$

$f'(0) = 8$

reciprocal

$g'(5)$

$1/8$

$f: (0, 5)$

$g: (5, 0)$

7b.) tan. line $f^{-1}(x)$ $x=3$

point

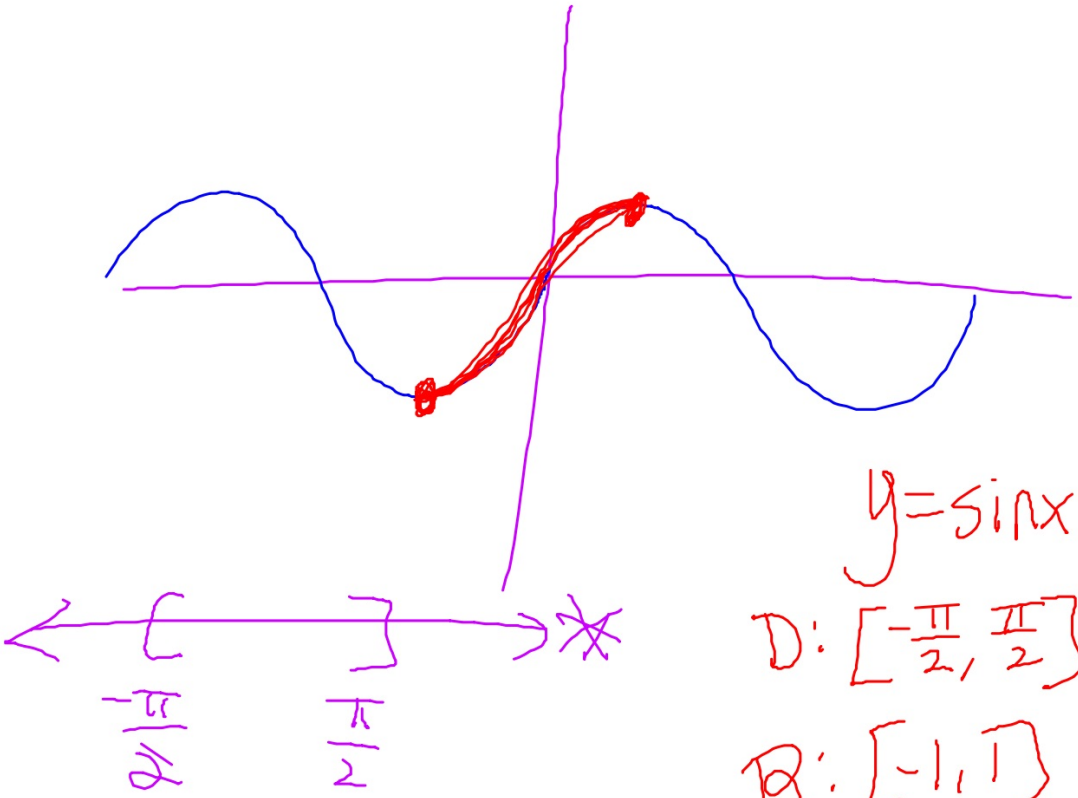
$f^{-1}(3, 1)$

$f'(1) = 2$

$f(1, 3)$

$y - 1 = \frac{1}{2}(x-3)$

slope

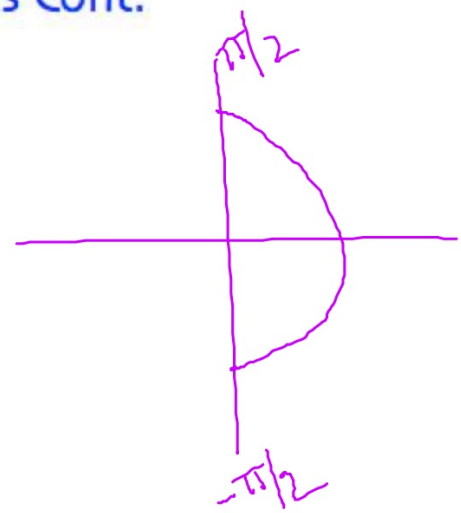


$y = \sin x$	$y = \sin^{-1} x$
D: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	D: $[-1, 1]$
R: $[-1, 1]$	R: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

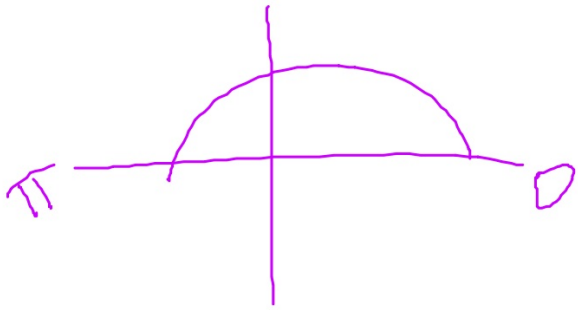
2.6 Derivatives of Inverse Functions Cont.

ex: Evaluate.

a) $\sin^{-1}\left(-\frac{1}{2}\right) = \frac{-\pi}{6}$

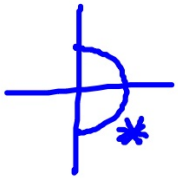


b) $\arccos(-1) = \pi$



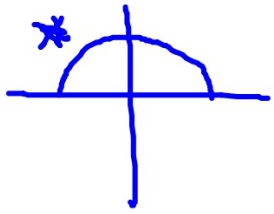
Belly functions	Sunset functions
arcsin x	arccos x
arccsc x	arcsec x
arctan x	arccot x

$\arccsc(-2)$



$-\frac{\pi}{6}$

$\arcsec(-2)$



$\frac{2\pi}{3}$

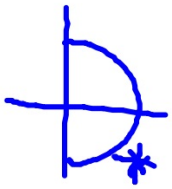
$\arccos\left(\frac{1}{2}\right)$

$\frac{\pi}{3}$

$\arcsin(0)$

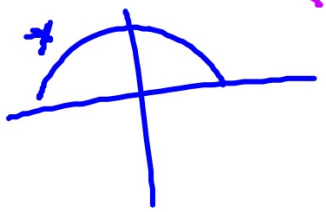
0

$\arctan(-1)$



$-\frac{\pi}{4}$

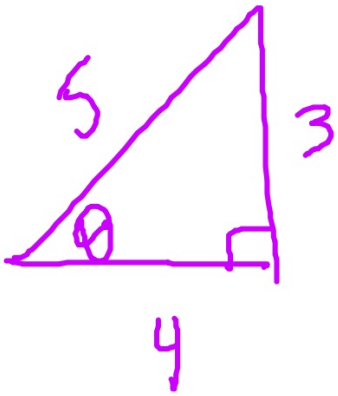
$\operatorname{arccot}(-\sqrt{3})$

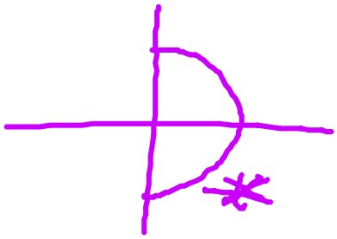


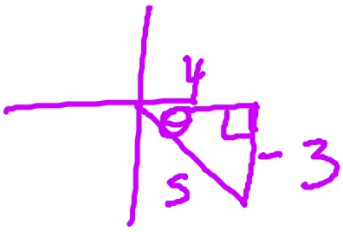
$\frac{5\pi}{6}$

ex: Evaluate.

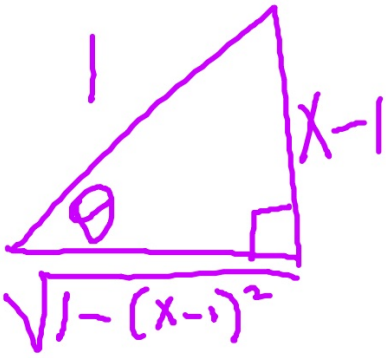
c) $\sin\left(\tan^{-1}\left(-\frac{3}{4}\right)\right) = \frac{-3}{5}$







d) $\cos\left(\arcsin\left(\frac{x-1}{1}\right)\right) = \sqrt{-x^2+2x}$



Derivatives of Inverse Trigonometric Functions

THEOREM 2.18 Derivatives of Inverse Trigonometric Functions

Let u be a differentiable function of x.

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

ex: Find the derivative.

a) $y = \sin^{-1}(2x)$

$$y' = \frac{2}{\sqrt{1-4x^2}}$$

$$y'\left(\frac{1}{4}\right) = \frac{2}{\sqrt{1-\frac{1}{4}}}$$

$$= \frac{2}{\sqrt{\frac{3}{4}}} = \frac{2}{\frac{\sqrt{3}}{2}}$$

$$= \frac{4}{\sqrt{3}}$$

ex: Find the derivative.

$$\left(e^{7x}\right)^2 = e^{14x}$$

b) $f(x) = \sec^{-1}\left(e^{7x}\right)$

$$f'(x) = \frac{7\cancel{e^{7x}}}{\cancel{1e^{7x}}\sqrt{e^{14x}-1}} = \frac{7}{\sqrt{e^{14x}-1}}$$

ex: Find an equation of the tangent line to the graph of f at the given point.

$$y = \arctan\left(\frac{x}{2}\right), \qquad x = -2$$

$\left(-2, \frac{-\pi}{4}\right)$

$$y' = \frac{\left(\frac{1}{2}\right)4}{\left(1 + \frac{x^2}{4}\right)4} = \frac{2}{4+x^2}$$

$$y + \frac{\pi}{4} = \frac{1}{4}(x+2$$

$$y'(-2) = \frac{\frac{1}{2}}{1+1} = \frac{1}{4}$$

Find the derivative.

$\sin(\underbrace{\arcsin \tfrac{1}{2}})$

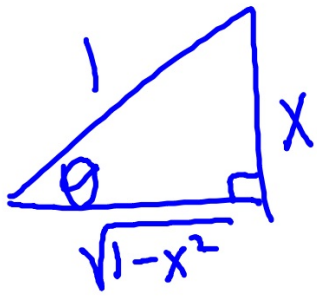
$\sin \frac{\pi}{6} = \frac{1}{2}$

y= cos(arcsinx)

$$y' = -\underbrace{\sin(\arcsin x)}_{x} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$y' = \frac{-x}{\sqrt{1-x^2}}$$

Find the derivative.



$y = \cos(\underbrace{\arcsin x})$

$$y = \frac{\sqrt{1-x^2}}{1}$$

$$y' = \frac{1}{2}(1-x^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{1-x^2}}$$