2.6 Derivatives of Inverse Functions

Review: Two functions are inverse functions if

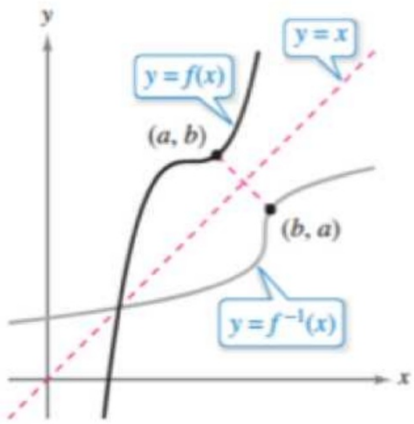
$$\left(f \circ f^{-1}\right)(x) =$$

and

$$\left(f^{-1} \circ f\right)(x) =$$

Inverse Properties

- The graphs of f and f^{-1} are reflections about the line y=x.
- The domain of f is the range of f^{-1}.
- The range of f is the domain of f^{-1}.
- A function has an inverse if and only if it is one-to-one.

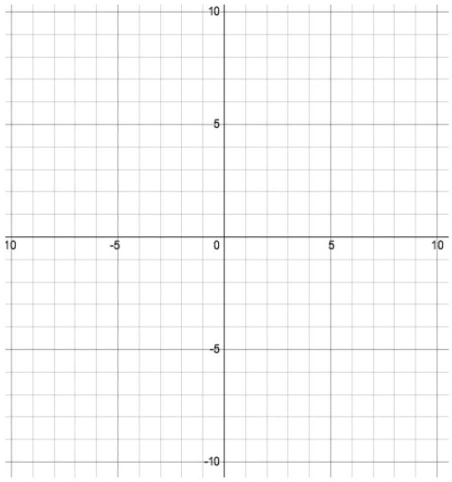


Derivative of an Inverse Function

$$\left(f \circ f^{-1}\right)(x) =$$

Sketch. Compare the slopes at f(1) and $f^{-1}(5)$

$$f(x) = x^3 + 4 \qquad f^{-1}(x) = \sqrt[3]{x-4}$$



1) $f(x) = 2x^3 + 3x$

Verify the function is 1:1 (monotonic: a function that is always increasing or always decreasing)

Find the derivative of the inverse at x = 5. $(f^{-1})'(5) =$ _______

2) $f(x) = x^3 - \frac{4}{x}$ $Domain: (0, \infty)$

f(x) and g(x) are inverses. Find g'(6)

ex: If $f(x) = x^5 + 2x^3 + x - 1$ and $(f \circ g)(x) = (g \circ f) = x$ find $g'(3)$.

3. 18% answered correctly

If $f(x) = x^3 + x$ and $h(x)$ is the inverse of $f(x)$, then $h'(2)$ is

A) $\frac{1}{13}$ B) $\frac{1}{4}$ C) 1 D) 4 E) 13

4. 14% answered correctly

Let f be a differentiable function such that $f(3) = 15, f'(3) = -8$, and $f'(6) = -2$, $f(6) = 3$
The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

A) $-\frac{1}{2}$ B) $-\frac{1}{8}$ C) $\frac{1}{6}$ D) $\frac{1}{3}$ E) The value of $g'(3)$ cannot be determined from the information given.

6.

Suppose f is a one-to-one function, which is differentiable for all real numbers x. The table below gives some of the values of $f(x)$ and $f'(x)$:

x	$f(x)$	$f'(x)$
1	2	$\frac{7}{6}$
2	3	$\frac{7}{6}$
3	5	$\frac{19}{6}$
4	10	$\frac{43}{6}$

(a) Write an equation of the tangent line, T_1, to the function $f(x)$ at $x = 3$.

____

(b) Write an equation of the normal line, N_1, to the function $f(x)$ at $x = 3$.

____

(c) Write an equation of the tangent line, T_2, to the function $f^{-1}(x)$ at $x = 3$.

____

10. Mean Score 0.95

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)dt$. Find the value $w'(3)$.

(d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

2.6 Derivatives of Inverse Functions Cont.

ex: Evaluate.

a) $\sin^{-1}\left(-\frac{1}{2}\right)$

b) $\arccos(-1)$

ex: Evaluate.

c) $\sin\left(\tan^{-1}\left(-\frac{3}{4}\right)\right)$

d) $\cos\left(\arcsin\left(x-1\right)\right)$

Derivatives of Inverse Trigonometric Functions

THEOREM 2.18 **Derivatives of Inverse Trigonometric Functions**

Let u be a differentiable function of x.

$\frac{d}{dx}[\arcsin u] =$ $\frac{d}{dx}[\arccos u] =$

$\frac{d}{dx}[\arctan u] =$ $\frac{d}{dx}[\text{arccot } u] =$

$\frac{d}{dx}[\text{arcsec } u] =$ $\frac{d}{dx}[\text{arccsc } u] =$

***We will re-visit the proofs of these derivatives next week :)*

ex: Find the derivative.

a) $y = \sin^{-1}(2x)$

ex: Find the derivative.

b) $f\left(x\right)=\sec^{-1}\left(e^{7x}\right)$

ex: Find an equation of the tangent line to the graph of f at the given point.

$$y = \arctan\left(\frac{x}{2}\right), \qquad x = -2$$