

$$23) f(x) = x^2(x-2)^4$$

$$f'(x) = \underline{x^2 \cdot 4(x-2)^3} + \underline{(x-2)^4 \cdot 2x}$$

$$= 2x(x-2)^3(2x + x-2)$$

$$= 2x(x-2)^3(3x-2)$$

$$55.) \quad f(x) = \frac{\cot x}{\sin x} = \frac{\frac{\cos x}{\sin x}}{\sin x} = \frac{\cos x}{\sin^2 x}$$

$(\sin x)^2$

$$\begin{aligned}f'(x) &= \frac{\sin^2 x (-\sin x) - \cos x \cdot 2(\sin x)^1 \cdot \cos x}{\sin^4 x} \\&= \frac{-\sin^3 x - 2\cos^2 x}{\sin^3 x}\end{aligned}$$

$$\begin{aligned}
 27.) \quad y &= x(x^2+1)^{-\frac{1}{2}} \\
 y' &= x \cdot -\frac{1}{2} (x^2+1)^{-\frac{3}{2}} \cancel{dx} + (x^2+1)^{-\frac{1}{2}} \cdot 1 \\
 &= (x^2+1)^{-\frac{3}{2}} \left(-x^2 + x^2 + 1 \right) \\
 &= \frac{1}{(x^2+1)^{\frac{3}{2}}}
 \end{aligned}$$

$$163.) \quad h'(1)$$

$$h(x) = f(g(x))$$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$f'(4) \left(-\frac{1}{2}\right)$$

$$(-1)\left(-\frac{1}{2}\right) = \frac{1}{2}$$

163 b.)

$s'(5)$

$$s(x) = g(f(x))$$

$$s'(x) = g'(f(x)) \cdot f'(x)$$

$$\cancel{g'(6)} \cdot (-1)$$



$$(67.) \sqrt{y} = \sin(\tan 2x)$$

$$y' = \cos(\tan 2x) \cdot \sec^2 2x \cdot 2$$

2.4 Chain Rule Cont.

ex: Differentiate.

a) $y = \csc(7x)$

b) $y = \frac{6}{(3x^2 - 5)^5}$

THEOREM 2.13 Derivative of the Natural Logarithmic Function

Let u be a differentiable function of x .

$$1. \frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$2. \frac{d}{dx}[\ln u] = \frac{1}{u} \cdot u'$$

ex: Differentiate.

a) $y = \ln(2x)$

$$y' = \frac{1}{2x} \cdot 2 = \frac{1}{x}$$

ex: Differentiate.

b) $y = x \ln x$

$$y' = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$y' = 1 + \ln x$$

ex: Differentiate.

c) $y = \ln \sqrt{x^2 + 8e^x}$

$$y = \frac{1}{2} \ln(x^2 + 8e^x)$$

$$y' = \frac{1}{2} \cdot \frac{1}{x^2 + 8e^x} \cdot (2x + 8e^x) = \frac{x + 4e^x}{x^2 + 8e^x}$$

$$y = \ln\left(\frac{\sqrt{x^2+1}}{x}\right)$$

$$y = \frac{1}{2} \ln(x^2 + 1) - \ln x$$

$$y' = \frac{1}{2} \cdot \frac{1}{x^2+1} \cdot 2x - \frac{1}{x}$$

$$y' = \frac{x}{x^2+1} - \frac{1}{x} = \frac{x - (x^2+1)}{x(x^2+1)} = \frac{-1}{x(x^2+1)}$$

THEOREM 2.14 Derivative Involving Absolute Value

If u is a differentiable function of x such that $u \neq 0$, then

$$\frac{d}{dx}[\ln|u|] = \frac{1}{u} \cdot u'$$

ex: Differentiate.

a) $f(x) = \ln|\csc x|$

ex: Differentiate.

$$\text{b) } f(x) = \ln \left| \frac{\cos x - 2}{e^x} \right|$$

$$\begin{aligned} f(x) &= \ln |\cos x - 2| - \ln |e^x| \\ &= \ln |\cos x - 2| - x \cdot \cancel{\ln e} \end{aligned}$$

$$f'(x) = \frac{1}{\cos x - 2} \cdot (-\sin x) - 1$$

$$= \frac{-\sin x}{\cos x - 2} - 1$$

ex: Differentiate.

c) $y = |\ln x|$

Definition of Logarithmic Function to Base a

If a is a positive real number ($a \neq 1$) and x is any positive real number, then the **logarithmic function to the base a** is denoted by $\log_a x$ and is defined as

$$\log_a x =$$

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a} = \frac{1}{\ln a} (\ln x)$$

ex: Differentiate.

a) $y = \log_7 x$

$$y' = \frac{1}{\ln 7} \cdot \frac{1}{x}$$

$$\boxed{y = \log_a u}$$
$$\boxed{y' = \frac{1}{\ln a} \cdot \frac{1}{u} \cdot u'}$$

ex: Differentiate.

b) $y = \log_6 \frac{x\sqrt{x-2}}{5}$

Derivative of the Exponential Function

Let u be a differentiable function of x .

$$\frac{d}{dx}[a^x] = \ln a \cdot a^x \quad \frac{d}{dx}[e^x] = e^x$$

$$\frac{d}{dx}[a^u] = \ln a \cdot a^u \cdot u' \quad \frac{d}{dx}[e^u] = e^u \cdot u'$$

ex: Differentiate.

a) $y = 3^x$

$$y' = \ln 3 \cdot 3^x$$

b) $y = e^{5x}$

$$\begin{aligned}y' &= e^{5x} \cdot 5 \\y' &= 5e^{5x}\end{aligned}$$

ex: Differentiate.

c) $y = 5^{\ln x}$

$$y' = \ln 5 \cdot 5^{\ln x} \cdot \frac{1}{x} = \frac{\ln 5 \cdot 5^{\ln x}}{x}$$

d) $f(x) = 4^{\log_4 x} = x$

$$f'(x) = 1$$

ex: Differentiate.

$$e) y = \frac{3}{e^x} = 3e^{-x}$$

$$y' = 3 \cdot e^{-x}(-1) = -3e^{-x} \text{ or } -\frac{3}{e^x}$$

$$f) y = \ln\left(\frac{1+e^x}{1-e^x}\right) = \ln(1+e^x) - \ln(1-e^x)$$

$$y' = \frac{1}{1+e^x} \cdot e^x - \frac{1}{1-e^x}(-e^x) = \frac{e^x}{1+e^x} + \frac{e^x}{1-e^x}$$

ex: Differentiate.

g) $y = e^x \ln x$ factor y'

$$\begin{aligned}y' &= e^x \cdot \frac{1}{x} + \ln x \cdot e^x \\&= e^x \left(\frac{1}{x} + \ln x \right) = e^x \left(\frac{1+x\ln x}{x} \right)\end{aligned}$$

h) $f(x) = \ln e^{777x} = 777x$

$$f'(x) = 777$$

ex: Write the equation of the tangent line to

$$y = e^{4x} \text{ at } x = \ln 2.$$

$$e^{4\ln 2}$$

$$e^{\ln 16}$$

$$y' = e^{4x} \cdot 4$$

$$y'(\ln 2) = 4e^{\ln 16} = 64$$

$$(\ln 2, 16)$$

$$y - 16 = 64(x - \ln 2)$$

integer

integer

$$\begin{aligned} 3\ln 2 \\ = \ln 8 \end{aligned}$$

ex: Write an equation of the tangent line to $y = \frac{\ln x}{4x}$ at $x=1$.

ex: Write an equation of the normal line to $y = \frac{\ln x}{4x}$ at $x=1$. $(1, 0)$

$$y' = \frac{4x \cdot \frac{1}{x} - \ln x \cdot 4}{16x^2}$$
$$y' = \frac{4 - 4\ln x}{16x^2}$$
$$y'(1) = \frac{1}{4}$$
$$y - 0 = -4(x - 1)$$

Summary of Differentiation Rules

General Differentiation Rules Let u and v be differentiable functions of x .

Constant Rule:

$$\frac{d}{dx}[c] = 0, c \text{ is a real number.}$$

Constant Multiple Rule:

$$\frac{d}{dx}[cu] = cu', c \text{ is a real number.}$$

Product Rule:

$$\frac{d}{dx}[uv] = uv' + vu'$$

Chain Rule:

$$\frac{d}{dx}[f(u)] = f'(u)u'$$

(Simple) Power Rule:

$$\frac{d}{dx}[x^n] = nx^{n-1}, \frac{d}{dx}[x] = 1, n \text{ is a rational number.}$$

Sum or Difference Rule:

$$\frac{d}{dx}[u \pm v] = u' \pm v'$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$$

General Power Rule:

$$\frac{d}{dx}[u^n] = nu^{n-1}u', n \text{ is a rational number.}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

Derivatives of Exponential and Logarithmic Functions

$$\frac{d}{dx}[e^x] = e^x$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0$$

$$\frac{d}{dx}[a^x] = (\ln a)a^x,$$

$$\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x},$$

a is a positive real number ($a \neq 1$).

a is a positive real number ($a \neq 1$).

ex: Find the derivative.

a) $y = 5x - 1$

d) $f(x) = \frac{e^{7x}}{3^x}$

b) $f(x) = x \log_5 x$

e) $y = \ln(\sec 2x)$

c) $g(x) = \cos^2 x$

f) $y = e^5 - 5e^4 + 7$

a) $y = 5x - 1$

$$y' = 5$$

$$\text{b) } f(x) = x \log_5 x$$

$$\begin{aligned}f'(x) &= x \cdot \frac{1}{\ln 5} \cdot \frac{1}{x} + \log_5 x \cdot 1 \\&= \frac{1}{\ln 5} + \log_5 x \\&= \frac{1}{\ln 5} + \frac{\ln x}{\ln 5}\end{aligned}$$

$$c) g(x) = \cos^2 x = (\cos x)^2$$

$$g'(x) = 2(\cos x)'(-\sin x)$$

$$\text{d) } f(x) = \frac{e^{7x}}{3^x}$$

$$f'(x) = \frac{3^x \cdot 7e^{7x} - e^{7x} \cdot \ln 3 \cdot 3^x}{3^{2x}}$$

$$\text{e) } y = \ln(\sec 2x)$$

$$y' = \frac{1}{\sec 2x} \cdot \cancel{\sec 2x} \tan 2x \cdot 2$$

$$f) \ y = e^5 - 5e^4 + 7$$

$$y' = \emptyset$$

ex:

If $f(x) = \sin(\ln(2x))$, then $f'(x) =$

(A) $\frac{\sin(\ln(2x))}{2x}$

(B) $\frac{\cos(\ln(2x))}{x}$

(C) $\frac{\cos(\ln(2x))}{2x}$

(D) $\cos\left(\frac{1}{2x}\right)$



ex:

Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

- (A) -0.701
- (B) -0.567
- (C) -0.391
- (D) -0.302
- (E) -0.258

FR 12

Let f be the function given by $f(x) = \ln \frac{x}{x-1}$.

- (a) What is the domain of f ?
- (b) Find the value of the derivative of f at $x = -1$.