# 2.4 Chain Rule

Outer function

$$y = f(g(x)) = f(u)$$

Inner function

**THEOREM 2.11 The Chain Rule**

If $y = f(u)$ is a differentiable function of $u$ and $u = g(x)$ is a differentiable function of $x$, then $y = f(g(x))$ is a differentiable function of $x$ and

$$\frac{d}{dx}[f(g(x))] =$$

ex: Which function(s) are good candidates for the chain rule?

- $y = \sin\left(x^2 + 1\right)$
- $y = e^{2x}$
- $y = \ln\left(x - 5\right)$
- $f\left(x\right) = \sin x \cos x$
- $y = \dfrac{5}{x + 1}$

ex: Find the derivative.

a) $y = \sin 2x$

b) $y = \sin(\cos x)$

ex: Find the derivative.

c) $y = \sqrt{e^x + 1}$

d) $y = \cos^4 x$

ex: Find the derivative.

e) $y = \tan^2 x$

f) $y = \tan x^2$

ex: Find the derivative.

g) $y = \sec 3x$

ex: Find the derivative.

h) $y = \left( \dfrac{3x-1}{x^2+2} \right)^3$

ex: Find the derivative.

i) $y = \dfrac{\sin^2 x}{4x - 1}$

ex: Find the derivative.

j) $f(x) = x^2\sqrt{1-x^2}$

ex: Find the derivative.

k) $y = \cos^4(3x)$

ex: $g(x) = \tan x,$ $\qquad \dfrac{d^2y}{dx^2} = ?$

ex: Write the equation of the tangent line to

$$y = \left(\frac{1}{x^2+9}\right)^2 \quad \text{at} \quad x = 1.$$

ex: Find the equation of the line that is tangent to $y = \sqrt{3x-1}$ and perpendicular to $3y + 2x = 3$.

ex:

| x | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 0 | 1 | −1 | 2 | 5 |
| 1 | −1 | 2 | 4 | 0 |
| 2 | 7 | 3 | 11 | 0.5 |

1. **Based on the values in the table above.**
   If $K(x) = f(g(x))$, then $K'(0) =$

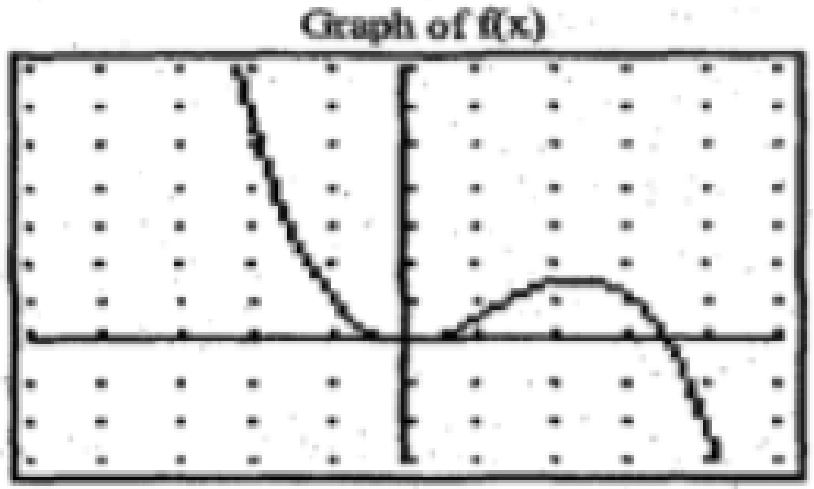
(A) 15 (B) 35 (C) −5 (D) −1 (E) 7

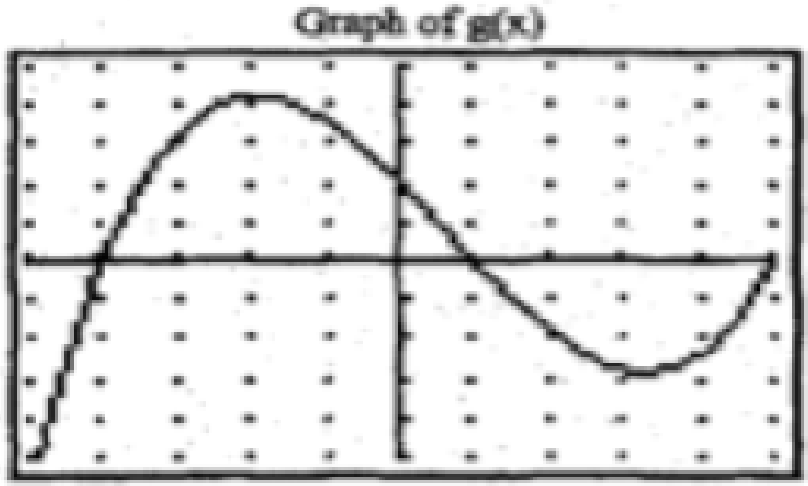
2. Let $g(x) = x \sin(x^2)$. Then $g'(x) =$

(A) $2x^2 \cos(x^2)$ (B) $-2x^2 \cos(x^2)$ (C) $2x^3 \cos(x^2)$
D) $2x^2 \cos(x^2) + \sin(x^2)$ (E) $-2x^2\cos(x^2) + \sin(x^2)$

ex:

Let f and g be the functions defined below




Let $h(x) = f(g(x))$ and let $k(x) = f(x^4)$

a) Evaluate h(-2), h(1), and h(2)

b) Is h'(-1) positive, negative, or equal to zero. Justify your answer.

c) Estimate the sign of h'(-2), h'(1), and h'(2).

ex: Find the derivative.

a) $y = \tan x \cot x$

b) $y = \dfrac{2}{(x-3)^4}$

c) $y = \left(\dfrac{x}{x^2+1}\right)^{-1}$

d) $y = \dfrac{\tan x}{\sin x - 1}$

e) $y = 2\sin x \cos x$

f) $y = \cot x \csc x$

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ex: Find the indicated derivative at the given point using your calculator.

a) $\left.\frac{d}{dx}(\ln x)\right|_{x=8}$

b) $f(x)=\tan^{-1}x, \quad f'(5)$

c) $y=\frac{3x+2}{\sqrt{x^2+5}}, \quad \left.\frac{d^2y}{dx^2}\right|_{x=\frac{1}{2}}$

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Let $f$ be the function given by $f(x) = \dfrac{x}{\sqrt{x^2 - 4}}$.

(a) Find the domain of $f$.

(b) Write an equation for each vertical asymptote to the graph of $f$.

(c) Write an equation for each horizontal asymptote to the graph of $f$.

(d) Find $f'(x)$.