2.3 Product & Quotient Rules

ex: Find the derivative.

a) $y=(x+1)(x^2-3)$

b) $g(x)=\dfrac{2x^2-1}{x}$

c) $y = e^x \sin x$

d) $f(x) = \dfrac{x^2 + 1}{2x - 1}$

THEOREM 2.8 The Product Rule

The product of two differentiable functions f and g is itself differentiable. Moreover, the derivative of fg is

$$\frac{d}{dx}[f(x)g(x)] =$$

ex: Which function(s) are good candidates for the product rule?

- $f(x) = 4\sin x$
- $y = xe^x$
- $y = (x+1)(x^2-3)$
- $y = 2x^3$
- $g(x) = \sin 2x$

ex: Find the derivative.

a) $y = xe^x$

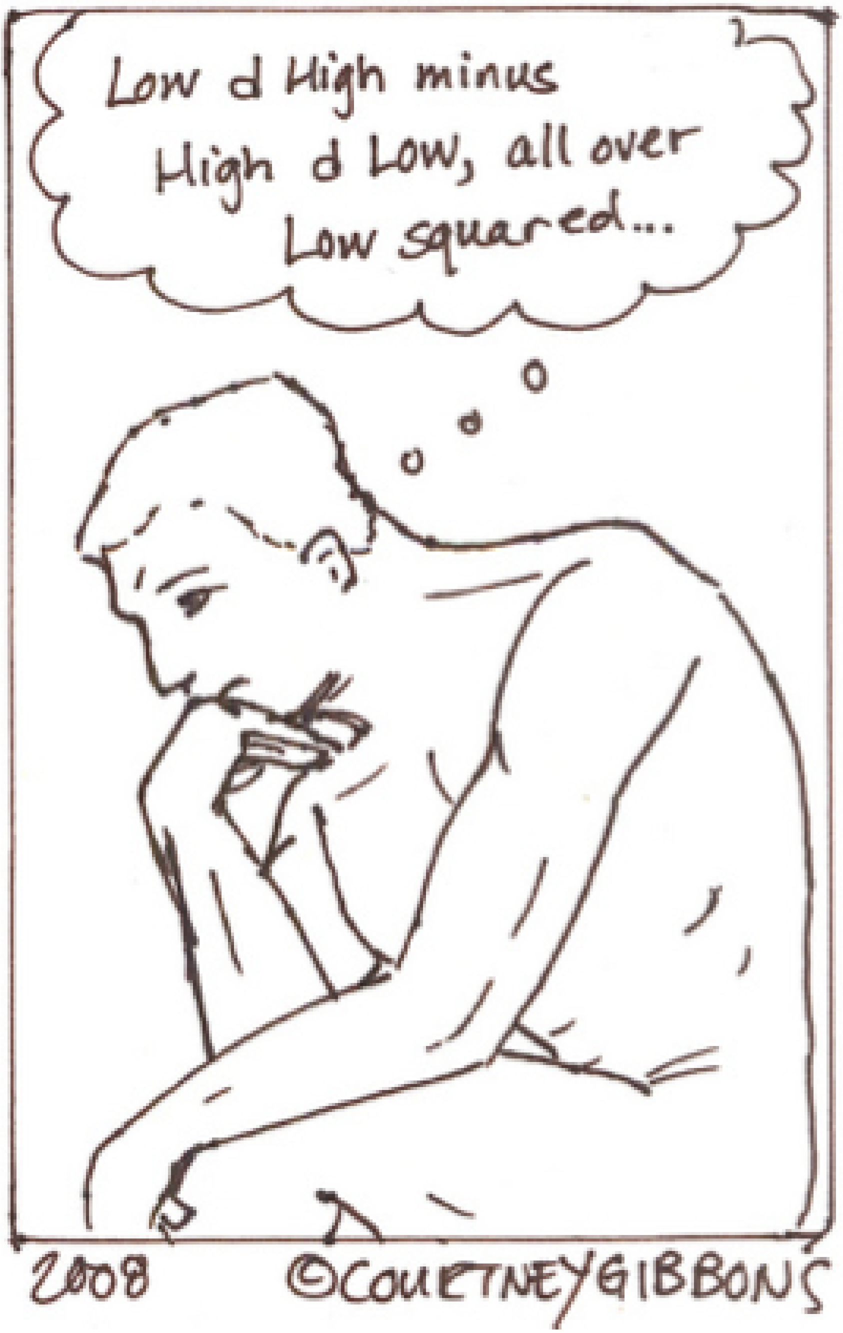
b) $g(x) = \sin 2x$

THEOREM 2.9 The Quotient Rule

The quotient f/g of two differentiable functions f and g is itself differentiable at all values of x for which $g(x) \neq 0$.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] =$$

Remembering The Quotient Rule...



ex: Which function(s) are good candidates for the quotient rule?

- $g(x) = \dfrac{5}{x}$
- $y = \dfrac{x}{5}$
- $y = \dfrac{e^x}{x+1}$
- $f(x) = \dfrac{x+1}{x}$
- $y = \tan x$

ex: Find the derivative.

a) $y = \dfrac{e^x}{x+1}$

ex: Find the derivative.

b) $f(x) = x^3\left(1 - \dfrac{2}{x+1}\right)$

ex: Find the derivative.

c) $f(x)=\dfrac{3-\dfrac{1}{x+5}}{x-1}$

ex: Find the derivative.

c) $y = \tan x$

ex: Find the derivative.

d) $y = \cot x$

ex: Find the derivative.

e) $y = \sec x$

ex: Find the derivative.

f) $y = \csc x$

Trigonometric Derivatives

$$\frac{d}{dx}[\sin x] =$$

$$\frac{d}{dx}[\csc x] =$$

$$\frac{d}{dx}[\cos x] =$$

$$\frac{d}{dx}[\sec x] =$$

$$\frac{d}{dx}[\tan x] =$$

$$\frac{d}{dx}[\cot x] =$$

Remembering The Derivatives of Tangent, Cotangent, Secant and Cosecant...

*MEMORIZE THIS CHART

$\tan x$	$\sec x$	$\sec x$
$\cot x$	$\csc x$	$-\csc x$

ex: Find the equation of the tangent line to $y = x^2 \sec x$ at $x = \frac{\pi}{4}$.

ex: Find the point(s), if any, at which $f(x) = \dfrac{x^2}{x^2+1}$ has a horizontal tangent.

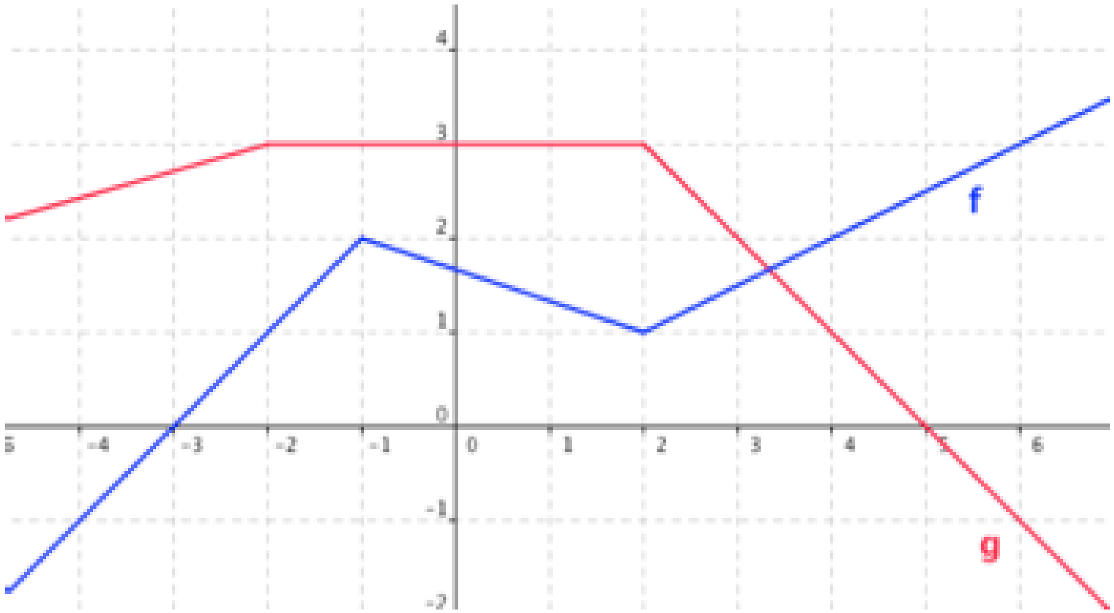
ex:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	–1	2	5
1	–1	2	4	0
2	7	3	11	0.5

Based on the values in the table above, if $H(x) = ef(x) + \pi x$, then $H'(0) =$

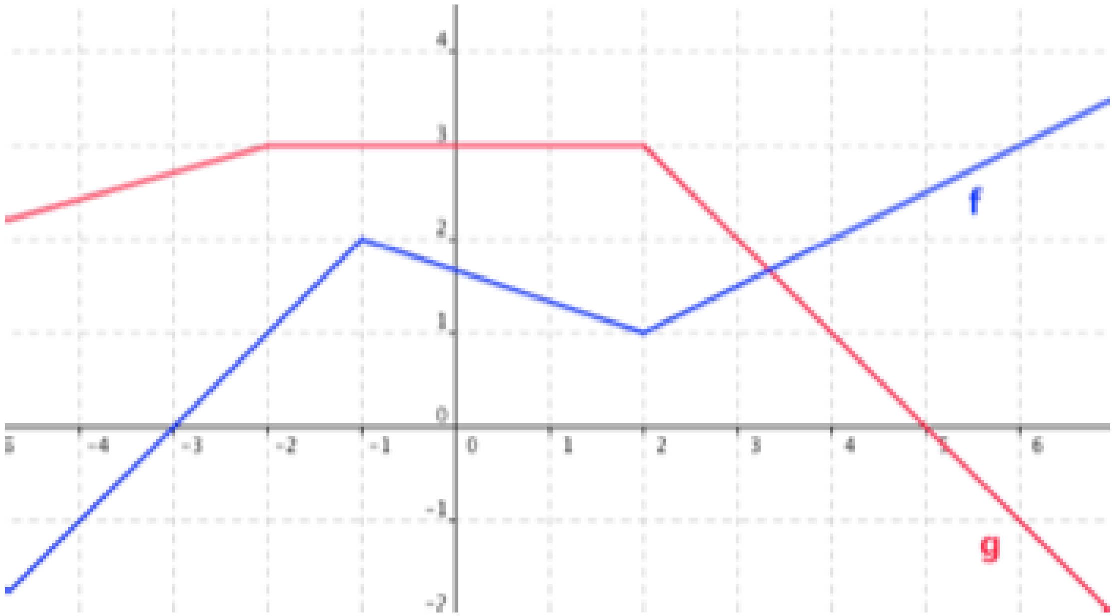
(A) $\pi - e$ (B) $e^x + \pi x$ (C) $e + \pi$ (D) e (E) $e^{-1} + \pi$

ex: Let $p(x) = f(x)g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$.

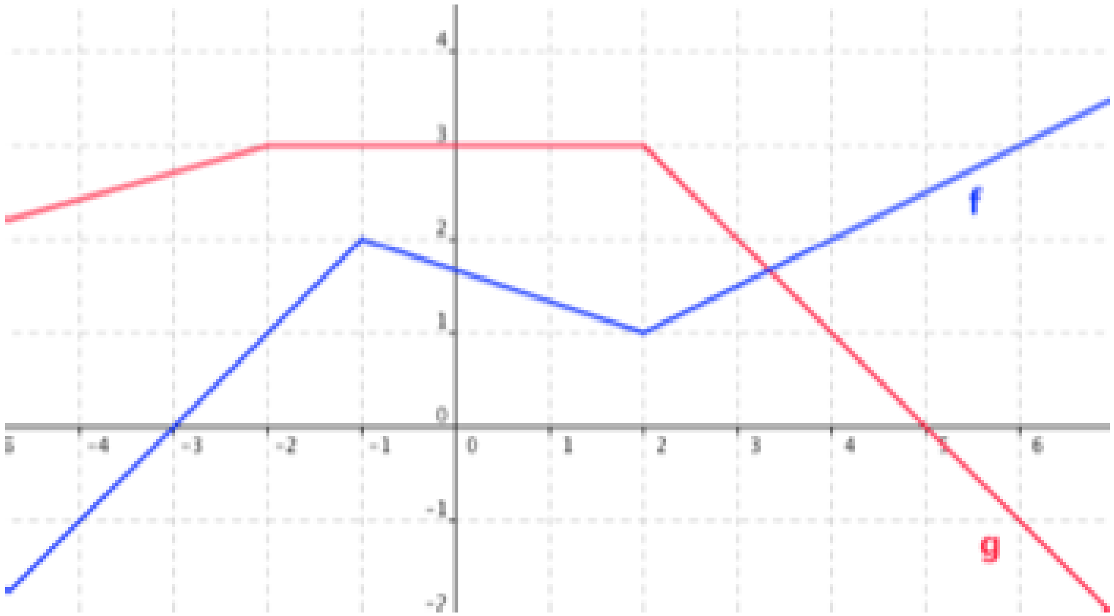


1. $p'(x)$

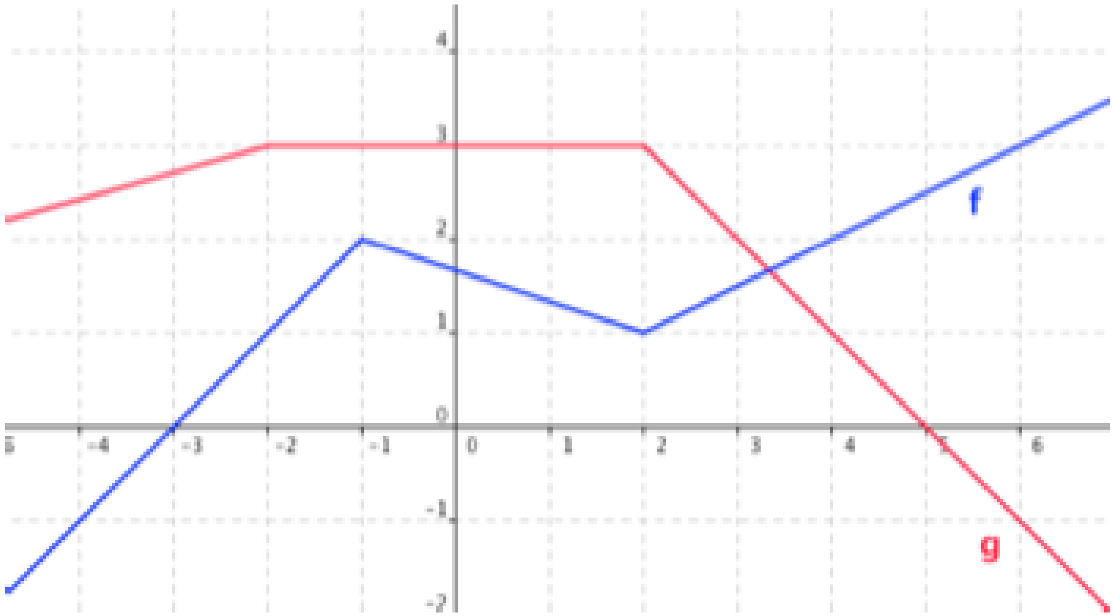
*See printout.



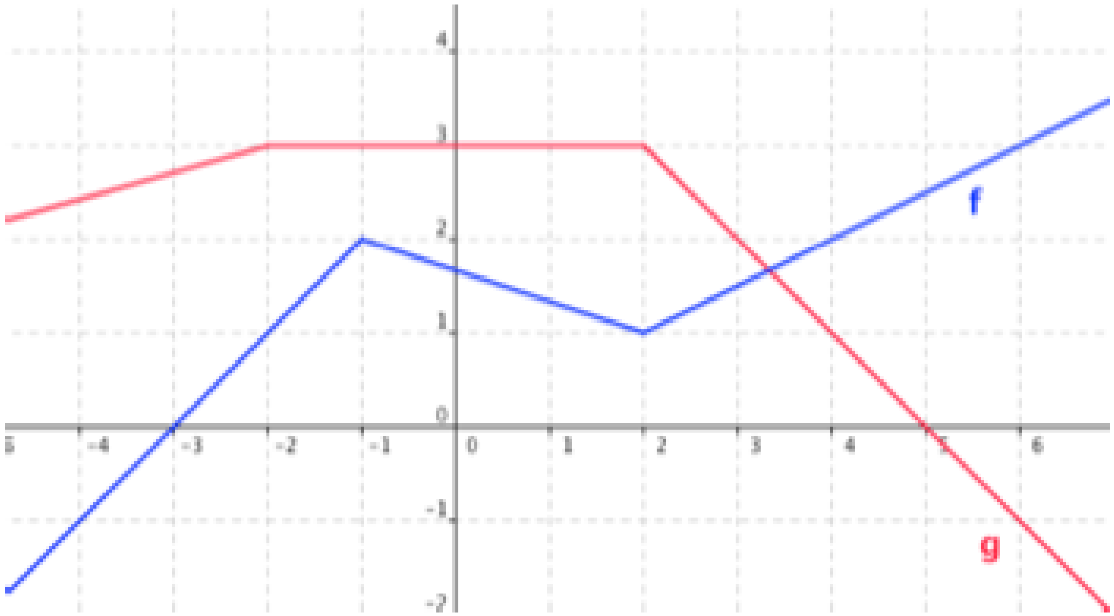
2. $p'(4)$



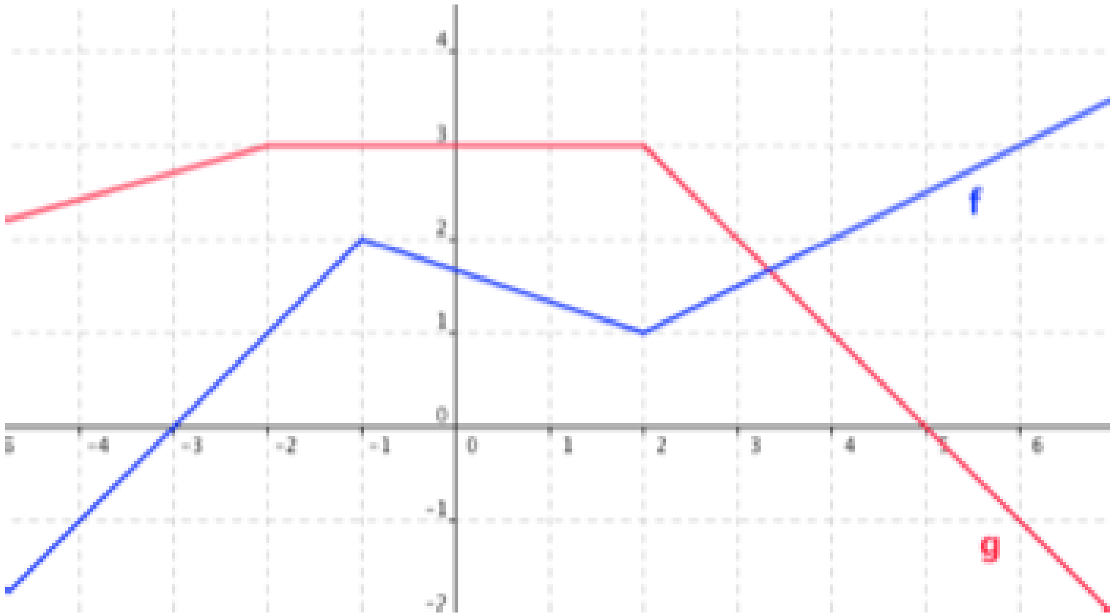
3. $p'(-1)$



4. $q'(x)$



5. $q'(-2)$



6. $q'(6)$

Higher Order Derivatives

First derivative:	y',	$f'(x)$,	$\dfrac{dy}{dx}$,	$\dfrac{d}{dx}[f(x)]$,	$D_x[y]$
Second derivative:	y'',	$f''(x)$,	$\dfrac{d^2y}{dx^2}$,	$\dfrac{d^2}{dx^2}[f(x)]$,	$D_x^2[y]$
Third derivative:	y''',	$f'''(x)$,	$\dfrac{d^3y}{dx^3}$,	$\dfrac{d^3}{dx^3}[f(x)]$,	$D_x^3[y]$
Fourth derivative:	$y^{(4)}$,	$f^{(4)}(x)$,	$\dfrac{d^4y}{dx^4}$,	$\dfrac{d^4}{dx^4}[f(x)]$,	$D_x^4[y]$
$\vdots$					
nth derivative:	$y^{(n)}$,	$f^{(n)}(x)$,	$\dfrac{d^ny}{dx^n}$,	$\dfrac{d^n}{dx^n}[f(x)]$,	$D_x^n[y]$

ex: Find the indicated derivative.

a) $f(x) = 3x^4 + 2, \qquad f'''(x) = ?$

ex: Find the indicated derivative.

b) $y = \sin x$, $\qquad \dfrac{d^2 x}{dx^2} = ?$

ex: Find the indicated derivative.

c) $y = \sin x,$ $\qquad \dfrac{d^5 x}{dx^5} = ?$

ex: Find the indicated derivative.

d) $y = \sin x,\qquad \dfrac{d^{205}x}{dx^{205}} = ?$

ex: Find the indicated derivative.

e) $g(x)=\cos x,\qquad g^{(163)}(x)=?$

ex: Find the indicated derivative.

f) $y = e^x$, $\qquad y^{(1111)} = ?$

FR 5

Let f be the function given by $f(x) = \frac{2x-5}{x^2-4}$.

a. Find the domain of f.
b. Write an equation for each vertical and each horizontal asymptote for the graph of f.
c. Find $f'(x)$.
d. Write an equation for the line tangent to the graph of f at the point (0,f(0)).

FR 18

Let f be the function that is given by $f(x) = \dfrac{ax+b}{x^2-c}$ and that has the following properties.

(i) The graph of f is symmetric with respect to the y-axis.

(ii) $\lim_{x \to 2^+} f(x) = +\infty$

(iii) $f'(1) = -2$

(a) Determine the values of a, b, and c.

(b) Write an equation for each vertical and each horizontal asymptote of the graph of f.

(c) Sketch the graph of f in the xy-plane provided below.