

A	24
B	6
C	1
D	2
F	0

$$\text{II.) } \lim_{x \rightarrow -3^-} \frac{x}{\sqrt{x^2 - 9}} = -\infty$$

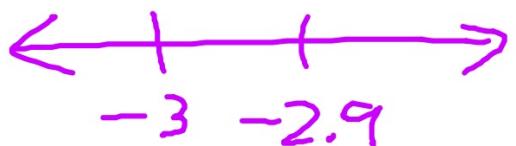
"-3.1"

$$\frac{-3.1}{\sqrt{(-3.1)^2 - 9}}$$

$$23.) \lim_{x \rightarrow 3} (2 - \lceil \frac{-x}{2.9} \rceil)$$

dne

:



justify

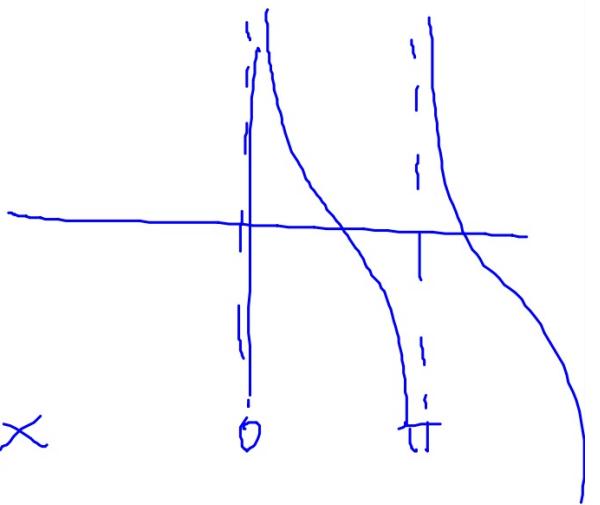
$$21.) \lim_{x \rightarrow 4^-} (5[x] - 7)$$

$$5[3.9] - 7$$

$$5 \cdot 3 - 7$$

$$19.) \lim_{x \rightarrow \pi} \cot x$$

$x \rightarrow \pi$
dne



$$\lim_{x \rightarrow \pi^-} \cot x \neq \lim_{x \rightarrow \pi^+} \cot x$$

$$-\infty \neq +\infty$$

$$27.) \lim_{x \rightarrow 2^-} \ln[x^2(3-x)]$$

$$\ln[4 \cdot 1]$$

$$\ln 4$$

1.5/1.6 Infinite Limits and Limits at Infinity

REVIEW:

Finding Horizontal Asymptotes - if $f(x)$ is a rational function...

$$f(x) = \frac{ax^n + \dots}{bx^m + \dots}$$

\leftarrow nth degree polynomial
 \leftarrow mth degree polynomial

BOBO

- [1] If $n < m$, then the x-axis is the horizontal asymptote.

$$y = 0$$

EATSDC

- [2] If $n = m$, then the horizontal asymptote is the line

$$y = \frac{a}{b}$$

BOTN

- [3] If $n > m$, then there is no horizontal asymptote.

NO HA

REMEMBER THE ACRONYM: BOBO BOTN EATSDC

*If $f(x)$ is not a rational function but comes in the form of a fraction compare the magnitudes of the numerator and denominator and use "BOBO."

Finding Vertical Asymptotes - Vertical asymptotes are vertical lines which correspond to the zeroes of the denominator of a simplified rational function. (They can also arise in other types of functions.)

*WATCH OUT FOR HOLES!

ex: State the horizontal and vertical asymptotes.

$$\text{a) } f(x) = \frac{x-1}{x^2 + 7x - 8} = \frac{1}{x+8}$$

VA: $x = -8$
HA: $y = 0$

b) $f(x) = \frac{x^2 - 4}{x - 5}$

VA: $x = 5$
HA: none

c) $f(x) = \frac{5x}{\sqrt{4x^2 + 1}}$

$$d) f(x) = \frac{\cos x}{2^x} \quad \text{cosx} \cdot 2^x$$

HA: $y = 0$
VA: none

$$e) f(x) = \frac{5 \cdot 3^x - 2}{1 \cdot 3^x}$$

VA: none
HA: $y = 5$
eats dc

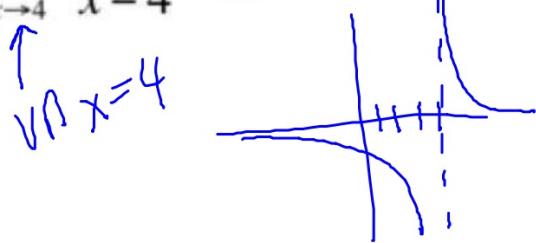
Infinite Limits:

$$\lim_{x \rightarrow c} f(x) = \infty$$

$$\lim_{x \rightarrow c} f(x) = -\infty$$

ex: Find the limit. If the limit does not exist, explain why.

a) $\lim_{x \rightarrow 4} \frac{1}{x-4}$ dne



$$\lim_{x \rightarrow 4^-} \frac{1}{x-4} \neq \lim_{x \rightarrow 4^+} \frac{1}{x-4}$$
$$-\infty \neq \infty$$

b) $\lim_{x \rightarrow 4} \frac{1}{(x-4)^2}$ ∞

c) $\lim_{x \rightarrow 4} \frac{1}{(x-4)^3}$ DNE

d) $\lim_{x \rightarrow 4} \frac{1}{(x-4)^4}$ 0

$$\text{e) } \lim_{x \rightarrow 7} \frac{x-9}{x-7}$$

DNE

$$\text{f) } \lim_{x \rightarrow 7} \frac{x-9}{(x-7)^2} -\infty$$

6.9
(-) 7.1
(-)

$$g) \lim_{x \rightarrow 6} \frac{x}{x^2 - 36} = \lim_{x \rightarrow 6} \frac{x}{(x+6)(x-6)} \quad DNE$$

$$h) \lim_{x \rightarrow 1} \frac{x-1}{x^2 - 7x + 6} = \lim_{x \rightarrow 1} \frac{1}{x-6} = -\frac{1}{5}$$

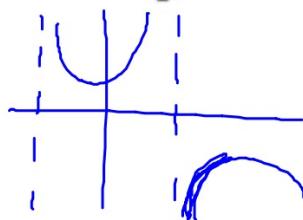
$$\text{i) } \lim_{x \rightarrow 6} \frac{x-1}{x^2 - 7x + 6}$$

$$\text{j) } \lim_{x \rightarrow 2} \frac{x^2 + 8x + 15}{x^2 + 3x - 10}$$

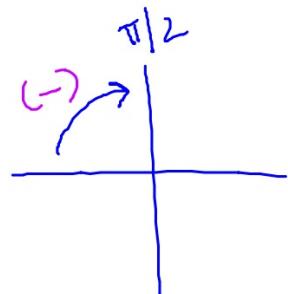
$$k) \lim_{x \rightarrow 0^-} \left(x^2 - \frac{1}{x} \right) = \lim_{x \rightarrow 0^-} x^2 - \lim_{x \rightarrow 0^-} \frac{1}{x} = 0 - (-\infty) = \infty$$

$$\lim_{x \rightarrow 0^-} \left(\frac{x^3 - 1}{x} \right) = \infty$$

$$l) \lim_{\substack{x \rightarrow \frac{\pi}{2}^+}} -2 \sec x = \infty$$



$$\lim_{\substack{x \rightarrow \frac{\pi}{2}^+}} \frac{-2}{\cos x}$$



$$m) \lim_{x \rightarrow -3^-} \frac{x}{\sqrt{x^2 - 9}} = -\infty$$

-3.1

In general, if $\lim_{x \rightarrow c} f(x) = \frac{n}{0}$, $n \neq 0$, then $f(x)$ must have a vertical asymptote at $x=c$.

- If the multiplicity of the factor that produces the vertical asymptote is odd, the limit will not exist. DNE
- If the multiplicity of the factor that produces the vertical asymptote is even, the limit exists and is either ∞ or $-\infty$. ∞

Limits at Infinity:

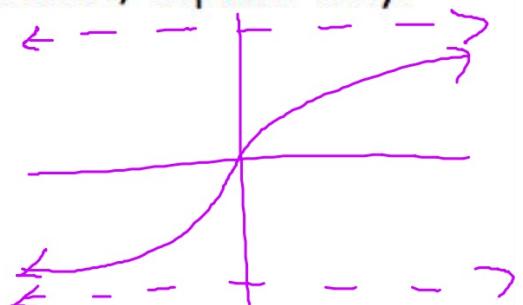
$$\lim_{x \rightarrow \infty} f(x) \quad \lim_{x \rightarrow -\infty} f(x)$$

*The existence or nonexistence of horizontal asymptotes will affect limits at infinity.

**KNOW YOUR LIBRARY OF FUNCTIONS!!!

ex: Find the limit. If the limit does not exist, explain why.

a) $\lim_{x \rightarrow \infty} \tan^{-1} x = \frac{\pi}{2}$



b) $\lim_{x \rightarrow \infty} \sin x$
dne (oscillating function)

c) $\lim_{x \rightarrow \infty} \frac{5x^2 - 4}{x^2 + 2} = 5$ has \int HA
 ∞ eats de.

d) $\lim_{x \rightarrow -\infty} \frac{5x^2 - 4}{x^2 + 2} = 5$

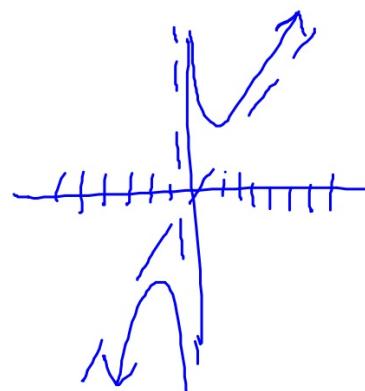
$$e) \lim_{x \rightarrow -\infty} \frac{5x-4}{x^2+2} = 0$$

$-\infty$
Bobo

$$\lim_{x \rightarrow -\infty} \frac{5x^2-4}{x+2} = -\infty$$

$$f) \lim_{x \rightarrow \infty} \frac{5x^2-4}{x+2} = \infty$$

Bobo



$$g) \lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$$

∞
Bobo

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$h) \lim_{x \rightarrow \infty} \frac{2^{-x}}{x^2} = \lim_{x \rightarrow \infty} \frac{\frac{1}{2^x}}{x^2} = 0$$

*Rational functions have
at most 1 HA*

* i) $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 + 2}}{5x - 3} = \frac{3}{5}$

X → ∞ eats d.c.

This is not rational

* j) $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^2 + 2}}{5x - 3} = -\frac{3}{5}$

X → -∞

$$\text{k) } \lim_{x \rightarrow \infty} \frac{\sqrt{5x^2 - 2x + 3}}{x - 4}$$

$$\text{l) } \lim_{x \rightarrow \infty} \frac{\sqrt{5x^2 - 2x + 3}}{4 - x}$$

Justifying Asymptotes

Horizontal Asymptotes:

If $f(x)$ has a horizontal asymptote at $y=c$ show

$$\lim_{x \rightarrow \infty} f(x) = C$$

or

$$\lim_{x \rightarrow -\infty} f(x) = C$$

Vertical Asymptotes:

If $f(x)$ has a vertical asymptote at $x=c$ show

$$\lim_{x \rightarrow c^-} f(x) = -\infty \text{ or } +\infty$$

(depends on $f(x)$)

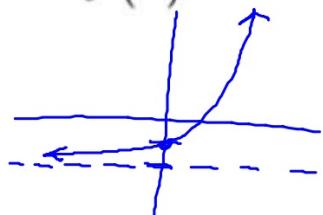
or

$$\lim_{x \rightarrow c^+} f(x) = -\infty \text{ or } +\infty$$

(depends on $f(x)$)

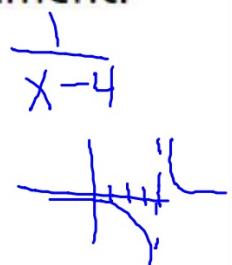
ex: State the horizontal and vertical asymptotes. Then justify your answers using an appropriate limit statement.

a) $f(x) = e^x - 2$



HA: $y = -2$

$$\lim_{x \rightarrow -\infty} f(x) = -2$$



b) $f(x) = \frac{\sqrt{6x^2 + 16}}{x - 2}$

VA: $x = 2$

$$\lim_{x \rightarrow 2^+} f(x) = \infty$$

HA: $y = \sqrt{6}$

$$\lim_{x \rightarrow \infty} f(x) = \sqrt{6}$$

HA: $y = -\sqrt{6}$

$$\lim_{x \rightarrow -\infty} f(x) = -\sqrt{6}$$

Put it all together...

ex: Find the limit or explain why it does not exist.

a) $\lim_{x \rightarrow 0} \frac{1}{3 + 5^{1/x}}$

b) $\lim_{x \rightarrow \infty} \frac{1}{3 + 5^{1/x}}$

c) $\lim_{x \rightarrow -\infty} \frac{1}{3 + 5^{1/x}}$

ex: Does the graph of y have any vertical or horizontal asymptotes? How do you know?

$$y = \frac{1}{3 + 5^{1/x}}$$

ex: If $\lim_{x \rightarrow 6^-} f(x) = \infty$, what can be said about the graph of $f(x)$?