1.5/1.6 Infinite Limits and Limits at Infinity

REVIEW:

<u>Finding Horizontal Asymptotes</u> - if f(x) is a rational function...

$$f(x) = \frac{ax^n + \dots}{bx^m + \dots}$$

← nth degree polynomial (numerator)

← mth degree polynomial (denominator)

1. If $n < m$, then the x-axis is the horizontal asymptote.
2. If $n = m$, then the horizontal asymptote is the line $y = \frac{a}{b}$
3. If $n > m$, then there is no horizontal asymptote.

REMEMBER THE ACRONYM: BOBO BOTN EATSDC

*If f(x) is not a rational function but comes in the form of a fraction compare the magnitudes of the numerator and denominator and use "BOBO."

Finding Vertical Asymptotes - Vertical asymptotes are vertical lines which correspond to the zeroes of the denominator of a simplified rational function. (They can also arise in other types of functions.)

*WATCH OUT FOR HOLES!

ex: State the horizontal and vertical asymptotes.

a) $f(x)=\frac{x-1}{x^2+7x-8}$

b) $f(x)=\dfrac{x^2-4}{x-5}$

c) $f(x)=\dfrac{5x}{\sqrt{4x^2+1}}$

d) $f(x)=\dfrac{\cos x}{2^x}$

e) $f(x)=\dfrac{5\cdot 3^x-2}{3^x}$

Infinite Limits:

$$\lim_{x \to c} f(x) = \infty \qquad \lim_{x \to c} f(x) = -\infty$$

ex: Find the limit. If the limit does not exist, explain why.

a) $\lim_{x \to 4} \frac{1}{x-4}$

e) $\lim_{x \to 7} \frac{x-9}{x-7}$

f) $\lim_{x \to 7} \frac{x-9}{(x-7)^2}$

g) $\lim_{x \to 6} \frac{x}{x^2 - 36}$

h) $\lim_{x \to 1} \frac{x - 1}{x^2 - 7x + 6}$

i) $\lim_{x \to 6} \frac{x-1}{x^2 - 7x + 6}$

j) $\lim_{x \to 2} \frac{x^2 + 8x + 15}{x^2 + 3x - 10}$

k) $\lim_{x \to 0}\left(x^2 - \frac{1}{x}\right)$

l) $\lim_{x \to \frac{\pi}{2}} -2\sec x$

m) $\lim_{x \to -3^-} \frac{x}{\sqrt{x^2 - 9}}$

In general, if $\lim_{x \to c} f(x) = \frac{n}{0}$, $n \neq 0$, then f(x) must have a

_______________________________ at x=c.

- If the multiplicity of the factor that produces the vertical asymptote is ____________, the limit will not exist.
- If the multiplicity of the factor that produces the vertical asymptote is ____________, the limit exists and is either ________ or ________.

Limits at Infinity:

$$\lim_{x \to \infty} f(x) \qquad \lim_{x \to -\infty} f(x)$$

*The existence or nonexistence of horizontal asymptotes will affect limits at infinity.

**KNOW YOUR LIBRARY OF FUNCTIONS!!!

ex: Find the limit. If the limit does not exist, explain why.

a) $\lim_{x \to \infty} \tan^{-1} x$

b) $\lim_{x \to \infty} \sin x$

c) $\lim_{x \to \infty} \frac{5x^2 - 4}{x^2 + 2}$

d) $\lim_{x \to -\infty} \frac{5x^2 - 4}{x^2 + 2}$

e) $\lim_{x \to -\infty} \frac{5x-4}{x^2+2}$

f) $\lim_{x \to \infty} \frac{5x^2-4}{x+2}$

g) $\lim_{x \to \infty} \frac{\sin x}{x}$

h) $\lim_{x \to \infty} \frac{2^{-x}}{x^2}$

i) $\lim_{x \to \infty} \frac{\sqrt{9x^2 + 2}}{5x - 3}$

j) $\lim_{x \to -\infty} \frac{\sqrt{9x^2 + 2}}{5x - 3}$

Justifying Asymptotes

<u>Horizontal Asymptotes</u>:

If f(x) has a horizontal asymptote at y=c show

or

Vertical Asymptotes:

If f(x) has a vertical asymptote at x=c show

or

ex: State the horizontal and vertical asymptotes. Then justify your answers using an appropriate limit statement.

a) $f(x) = e^x - 2$

b) $y = \dfrac{\sqrt{6x^2 + 16}}{x - 2}$