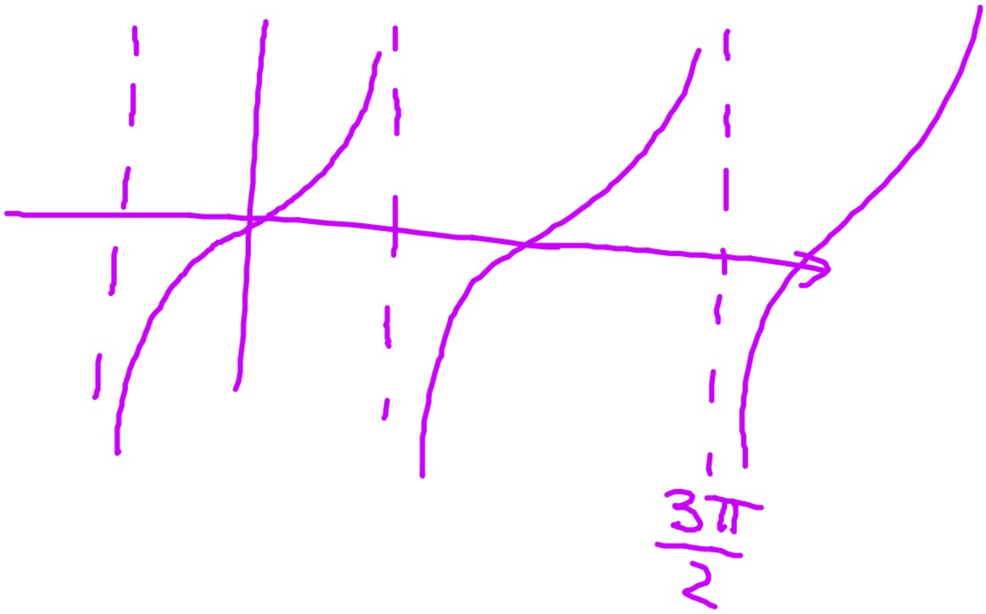
3.) $\lim_{x \to \frac{3\pi}{4}^+} \left(-2\tan 2x\right)$ ∞



17. $$\lim_{x\to -2^-} \frac{x+2}{x^2+x-2}$$

$$\lim_{x\to -2^-} \frac{\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$$

$$\frac{1}{-3}$$

5.) $\lim_{x \to \infty} x\cos\left(\frac{1}{x}\right)$

∞

8. $\lim_{x \to \infty} \frac{\sin x}{x} = 0$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

1982 $f(x) = \dfrac{x^3 - x}{x^3 - 4x} = \dfrac{\cancel{x}(x+1)(x-1)}{\cancel{x}(x+2)(x-2)}$

(b) $0 = (x+1)(x-1)$

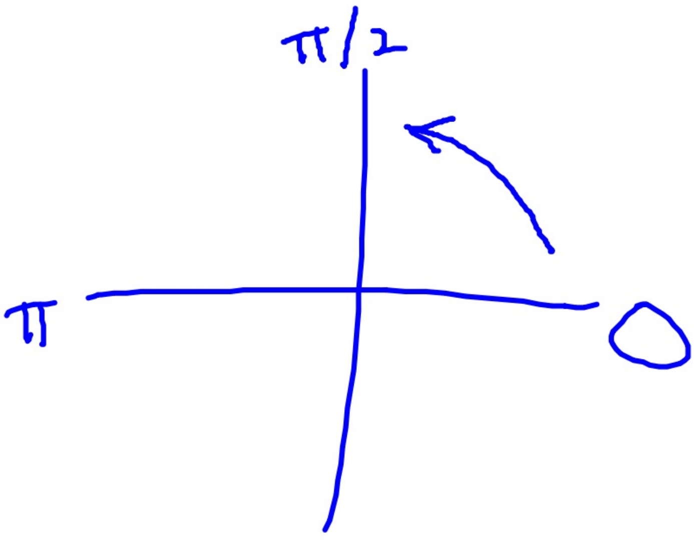
zeros

$x = -1, 1$

20.) $f(x) = \dfrac{x^2-1}{x^2+3x-4} = \dfrac{(x+1)\cancel{(x-1)}}{(x+4)\cancel{(x-1)}}$

VA	HA
$x = -4$	$y = 1$
$\lim_{x \to -4^-} f(x) = \infty$	$\lim_{x \to \infty} f(x) = 1$

10.) $\lim_{x \to \frac{\pi}{4}^-} 2\sec 2x = \infty$



Skip #65

1.4 Continuity At A Point & The Intermediate Value Theorem

ex: If $f(2)=4$, can you conclude anything about the limit of $f(x)$ as x approaches 2? Explain your reasoning.

ex: If the limit of $f(x)$ as x approaches 2 is 4, can you conclude anything about $f(2)$? Explain your reasoning.

Types of Discontinuities

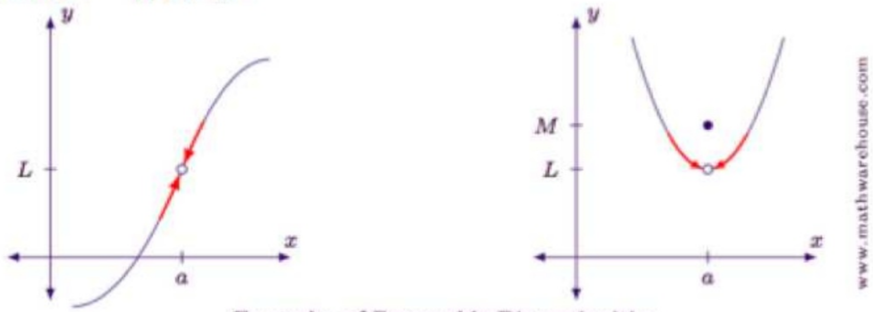
Skip 65

- Removable - holes

$$\frac{x^2-1}{x-1}$$

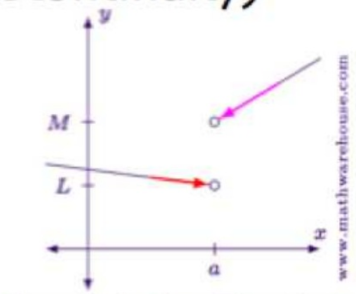
$x+1$

hole at $x=1$

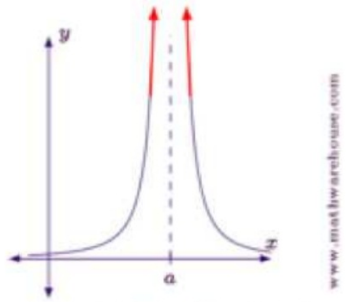


Examples of Removable Discontinuities

- Nonremovable - jumps, vertical asymptotes (a.k.a. infinite discontinuity)



Example of a Jump Discontinuity



Example of an Infinite Discontinuity

ex: At what x-values, if any, is f(x) discontinuous? For each discontinuity state the x-value, the type of discontinuity, and whether the discontinuity is removable or nonremovable.

$$f(x)=\frac{x^2-1}{x^2-4x+3} = \frac{x+1}{x-3}$$

Remov.
at $x=1$

nonremov.
$x=3$

Continuity At A Point, x=c

DEFINITION OF CONTINUITY

Continuity at a Point: A function f is **continuous at** c if the following three conditions are met.

1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$

$$\lim_{x \to c} f(x) = f(c)$$

Continuity on an Open Interval: A function is **continuous on an open interval** (a, b) if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is **everywhere continuous.**

ex: Is f(x) continuous at x=0? Justify your answer.

$$f(x)=\begin{cases} x+1, & x\le 0 \\ x^2+1, & x>0 \end{cases}$$

$\lim_{x\to 0} f(x) = f(0)$ ✓ Yes!

$1 = 1$

ex: Is g(x) continuous at x=3? Justify your answer.

$$g(x) = \frac{x^3 - 27}{x - 3}$$

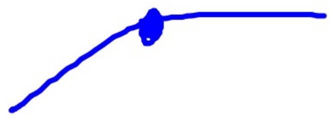
$$= \frac{\cancel{(x-3)}(x^2 + 3x + 9)}{\cancel{x-3}}$$

$$= x^2 + 3x + 9$$

disc. at $x-3$ (removable)

$\lim_{x \to 3} g(x) \neq g(3)$
(exists)

ex: Find the value of b so that the function f(x) is continuous everywhere.



$$f(x)=\begin{cases} x+3, & x\le 2 \\ bx+7, & x>2 \end{cases}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$$

$$\lim_{x\to 2^-} (x+3) = \lim_{x\to 2^+} (bx+7) = 5$$

$$5 = 2b+7$$

$$\boxed{-1 = b}$$

ex: Find the value of a so that the function h(x) is continuous everywhere.

$$h(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & x \neq a \\ 28, & x = a \end{cases} \longleftarrow (a, 28)$$

$$\lim_{x \to a} h(x) = h(a)$$

$$\lim_{x \to a} (x + a) = 28$$

$$2a = 28 \quad a = 14$$

$$f(x) = \begin{cases} Ax - B\ , & \text{if } x \le -1 \\ 2x^2 + 3Ax + B\ , & \text{if } -1 < x \le 1 \\ 4\ , & \text{if } x > 1 \end{cases}$$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\boxed{2 + 3A + B = 4} = 2 + 3A + B$

$\boxed{3A + B = 2}$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = f(-1)$

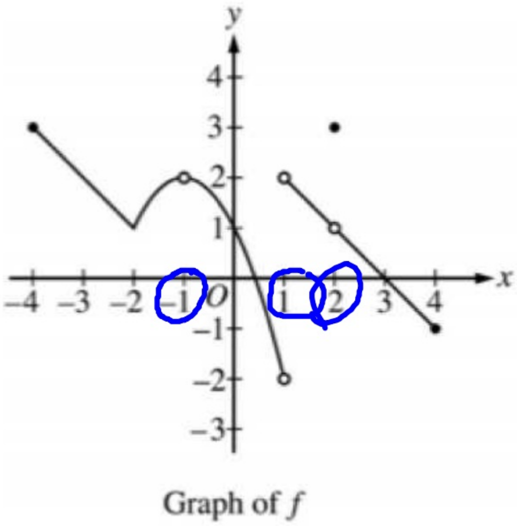
$\boxed{-A - B = 2 - 3A + B}$

$2A - 2B = 2$

$\boxed{A - B = 1}$

$$\begin{aligned} 3A + B &= 2 \\ A - B &= 1 \\ \hline 4A &= 3 \\ A &= 3/4 \\ B &= -1/4 \end{aligned}$$

ex:



Graph of f

The graph of the function f is shown in the figure above. For how many values of x in the open interval $(-4, 4)$ is f discontinuous?

(A) one

(B) two

(C) three

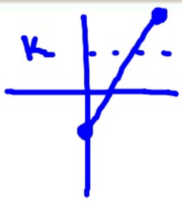
(D) four

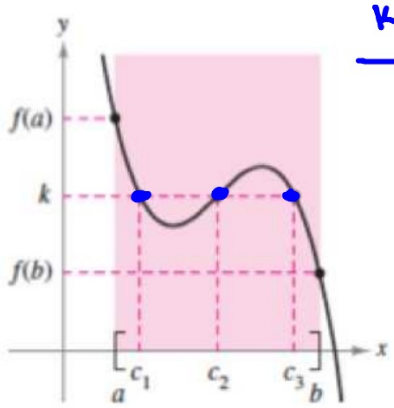
Intermediate Value Theorem

THEOREM 1.13 INTERMEDIATE VALUE THEOREM (IVT)

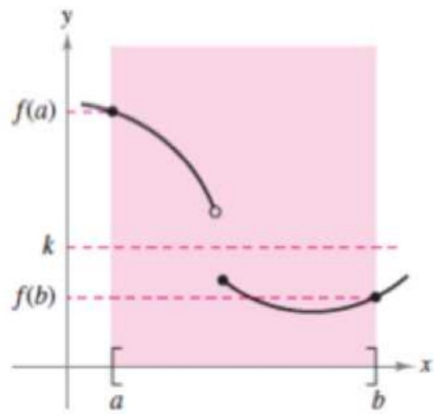
If f is continuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that

$$f(c) = k.$$



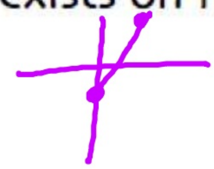


f is continuous on $[a, b]$.
[There exist three c's such that $f(c) = k$.]



f is not continuous on $[a, b]$.
[There are no c's such that $f(c) = k$.]

ex: Use the Intermediate Value Theorem to show a zero exists on f(x) on the given interval.



$$f(x) = x^3 + 2x - 1, \qquad [0,1]$$

$f(0) = -1$
$f(1) = 2$
$f(c) = 0$?

Since f(x) is continuous on [0,1] and f(0) < 0 < f(1), by IVT there exists at least 1 value c in (0,1) such that f(c) = 0.

continuous function

ex: Consider the table of values of f(x) given below.

x	0	2	3	10	20
$f(x)$	-2	3	4	20	-10

What is the least amount of time f(x)=15 on [0, 20]? Justify your answer.

$f(3)=4$ [3, 10]
$f(10)=20$

Since f(x) is continuous on [3, 10] and f(3) < 15 < f(10) by IVT there exists a value c in (3,10) such that f(c) = 15.

$f(10)=20$ [10, 20]
$f(20)=-10$

Since f(x) is continous on [10, 20] and f(20) < 15 < f(10), by IVT there exists a value c in (10,20) such that f(c) = 15.

Verify that IVT applies. Then find c such that f(c) = 4.

$$f(x) = x^3 - x^2 + x - 2 \qquad [0, 3]$$

$f(0) = -2$

$f(3) = 19$

Since f(x) is cont. [0,3] and f(0) < 4 < f(3) by IVT there exists a value c in (0,3) such that f(c) = 4.

$$x^3 - x^2 + x - 2 = 4$$

$x = 2$ ✓

$$x^3 - x^2 + x - 6 = 0$$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -6 \\ & & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & 0 \end{array}$$

$$x^2 + x + 3 = 0$$

$c = 2$