

$$65) \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - 2x - 2\Delta x + 1}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{(2x + \cancel{\Delta x} - 2)}{2x - 2}$$

$$\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad f(x) = \frac{1}{x+3}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\left(\frac{1}{x+\Delta x+3} - \frac{1}{x+3} \right)}{(\Delta x)} \quad f(x + \Delta x) = \frac{1}{x+\Delta x+3}$$

$$\lim_{\Delta x \rightarrow 0} \frac{x+3 - (x+\cancel{\Delta x}+3)}{\cancel{\Delta x}(x+3)(x+\Delta x+3)} = \lim_{\Delta x \rightarrow 0} \frac{-1}{(x+3)(x+\cancel{\Delta x}+3)}$$

$$= \frac{-1}{(x+3)^2}$$

$$79.) \lim_{x \rightarrow 0} \frac{\left(1 - \frac{1}{e^x}\right) \cdot e^x}{(e^x - 1) \cdot e^x}$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{(e^x - 1)e^x}$$

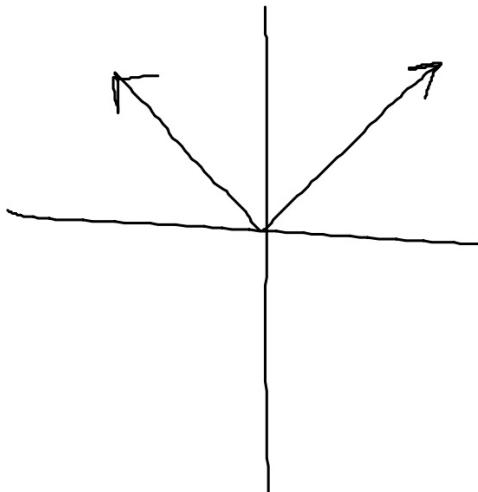
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1.4 One-Sided Limits

REVIEW: Rewrite each absolute value function as a piecewise function.

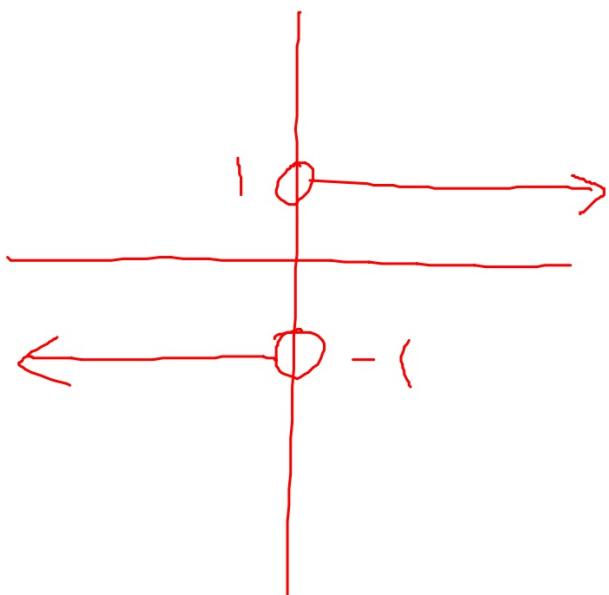
a) $y = |x|$

$$y = \begin{cases} -x, & x \leq 0 \\ x, & x > 0 \end{cases}$$



$$\text{b) } y = \frac{|x|}{x}$$

$$y = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \end{cases}$$



One-Sided Limits

$$\lim_{x \rightarrow c^-} f(x)$$

Left-Sided Limit

$$\lim_{x \rightarrow c^+} f(x)$$

Right-Sided Limit

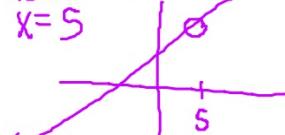
*Use the techniques you learned in 1.2 and 1.3 to find one-sided limits.

ex: Find the limit. If the limit does not exist, explain.

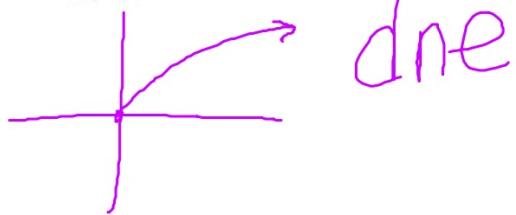
a) $\lim_{x \rightarrow 6^-} (x^2 - 21) = 36 - 21$
= 15

b) $\lim_{x \rightarrow 5^+} \frac{x^2 - 25}{x - 5} = \lim_{x \rightarrow 5^+} (x + 5) = 10$

hole@
 $x=5$



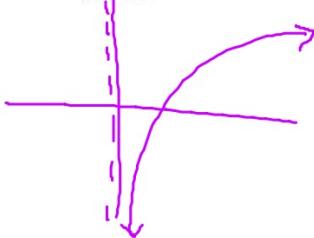
c) $\lim_{x \rightarrow 0^-} \sqrt{x}$



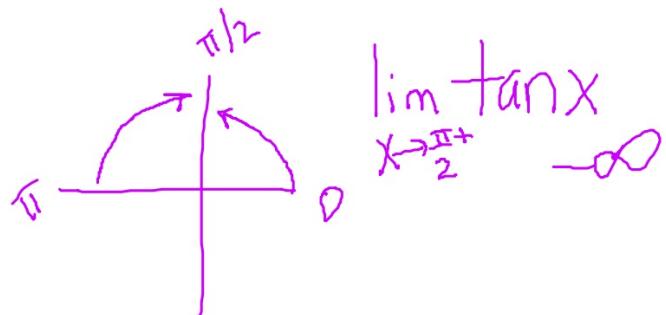
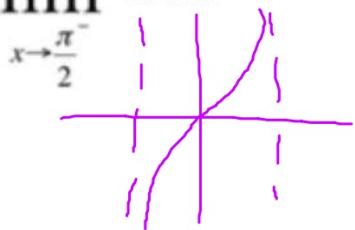
dne

The function ceases to exist to the left of zero.

d) $\lim_{x \rightarrow 0^+} \ln x$ $-\infty$



e) $\lim_{x \rightarrow \frac{\pi}{2}^-} \tan x$ ∞



ex: Given $f(x)$ find each limit. If the limit does not exist, explain.

$$f(x) = \begin{cases} x^2 + 4, & x > 5 \\ 2x - 3, & x \leq 5 \end{cases}$$

a) $\lim_{x \rightarrow 5} f(x)$ dne

$$\lim_{x \rightarrow 5^-} f(x) \neq \lim_{x \rightarrow 5^+} f(x)$$
$$7 \neq 29$$

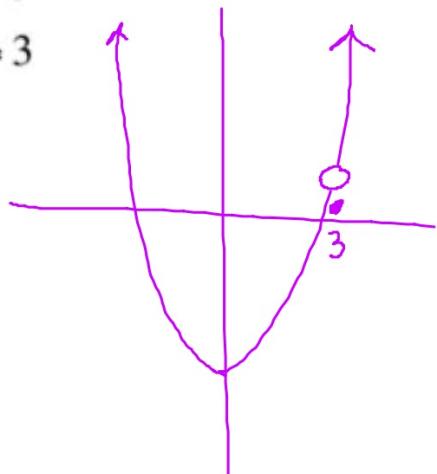
b) $\lim_{x \rightarrow 11} f(x) = 125$

ex: Given $h(x)$ find each limit. If the limit does not exist, explain.

hole @ $(3, 2)$

$$h(x) = \begin{cases} x^2 - 7, & x \neq 3 \\ 1, & x = 3 \end{cases}$$

a) $\lim_{x \rightarrow 3} h(x) = 2$

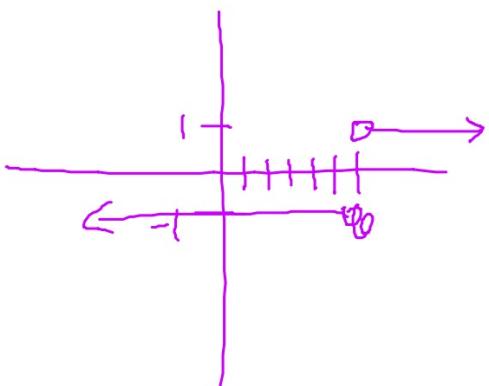


b) $\lim_{x \rightarrow 0} h(x) = -7$

ex: Find the limit. If the limit does not exist, explain.

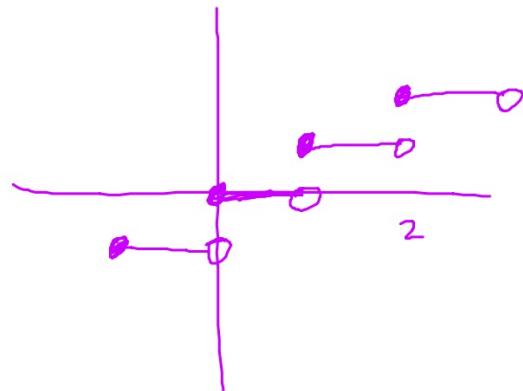
a) $\lim_{x \rightarrow 6} |x - 6| = \textcircled{0}$

b) $\lim_{x \rightarrow 6} \frac{|x - 6|}{x - 6}$ dne
let $f(x) = \frac{|x - 6|}{x - 6}$ $\lim_{x \rightarrow 6^-} f(x) + \lim_{x \rightarrow 6^+} f(x)$



c) $\lim_{x \rightarrow -5} \frac{|x+5|}{x-3} = 0$

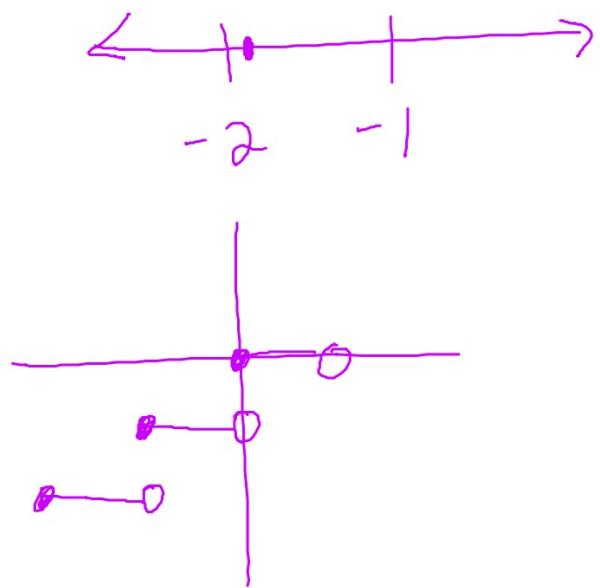
d) $\lim_{x \rightarrow 2} [x]$ due
justify...



e) $\lim_{x \rightarrow 2.3} [x] = 2$

$$f) \lim_{x \rightarrow -2^+} [x] = -2$$

$x \rightarrow -2^+$
↓
 -1.9



For a differentiable function f , let f^* be the function defined by

$$f^*(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{h}$$

(a) Determine $f^*(x)$ for $f(x) = x^2 + x$

$$f(x+h) = (x+h)^2 + x + h$$

(b) Determine $f^*(x)$ for $f(x) = \cos x$

$$f(x+h) = \cos(x+h) = \dots$$

(a)