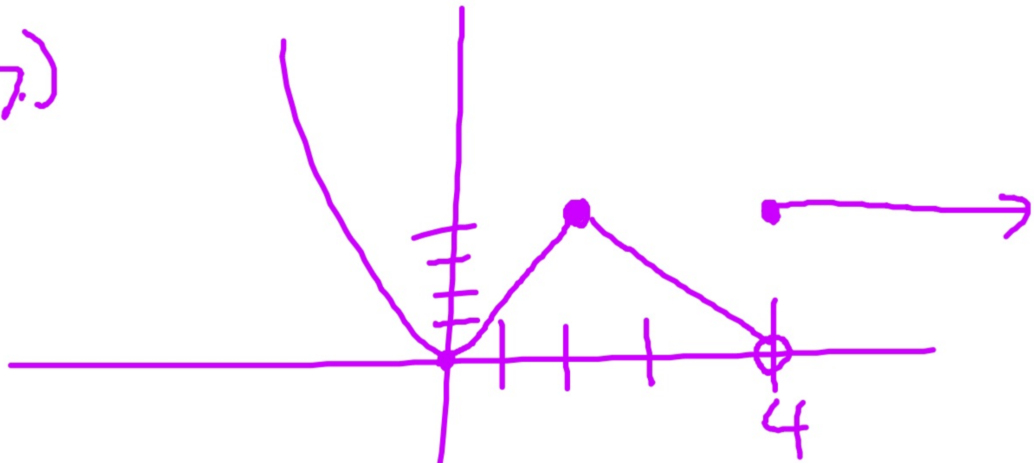
27.)



$$\lim_{x \to c} f(x)$$

exists except where $c = 4$

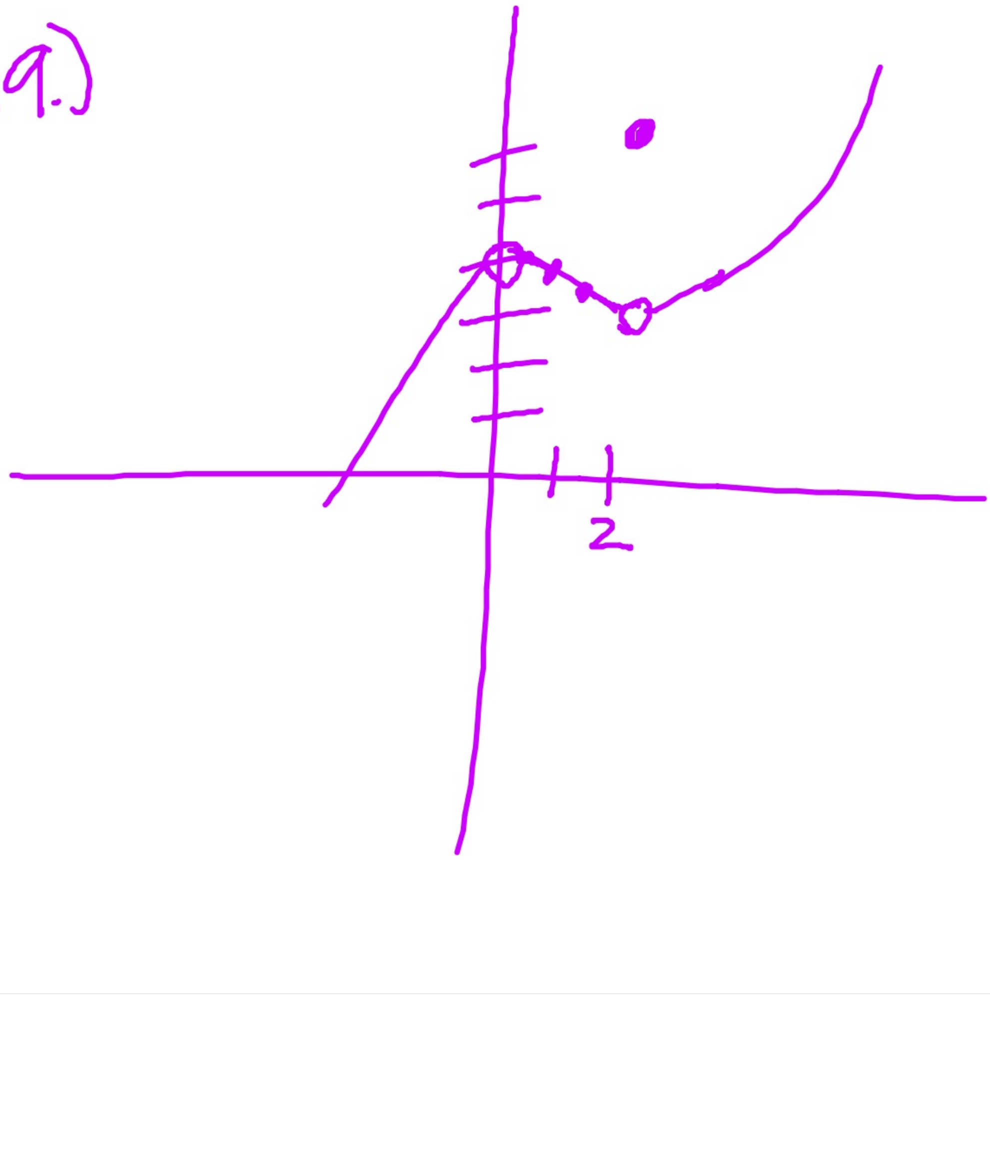
$\{c \mid c \neq 4\}$

31.) $\lim_{x \to 3} \tan \frac{\pi x}{4} = \tan \frac{3\pi}{4} = -1$

$$\lim_{x \to 1} \left(\ln 3x + e^{x} \right)$$

$$\ln 3 + e$$

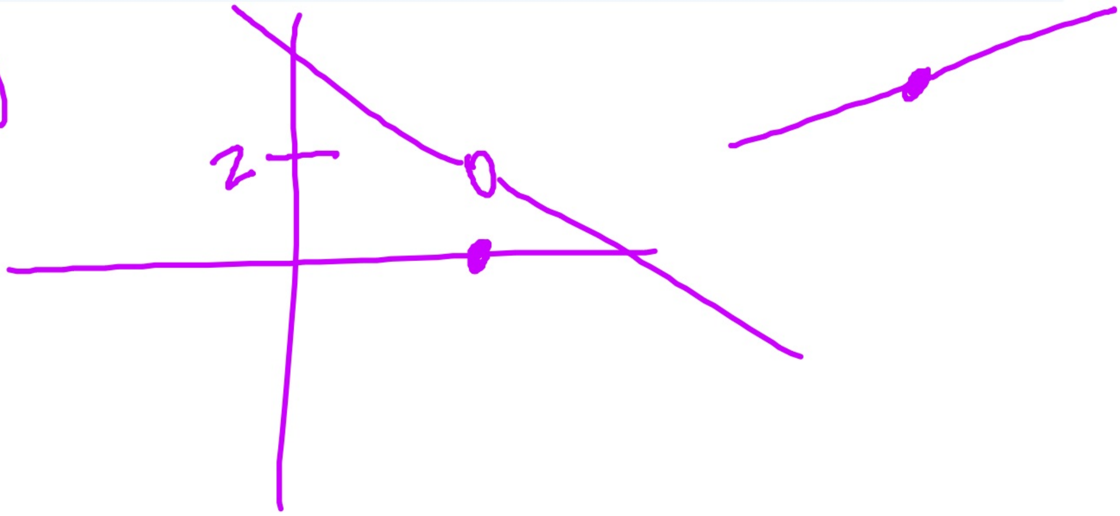
29.)



1.3 Evaluating Limits Analytically

- Evaluate a limit using properties of limits.
- Develop and use a strategy for finding limits.
- Evaluate a limit using dividing out and rationalizing techniques.
- Evaluate a limit using the Squeeze Theorem.

19.)



THEOREM 1.2 PROPERTIES OF LIMITS

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \to c} f(x) = L \qquad \text{and} \qquad \lim_{x \to c} g(x) = K$$

1. Scalar multiple: $\lim_{x \to c} [b f(x)] = bL$

2. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$

3. Product: $\lim_{x \to c} [f(x)g(x)] = LK$

4. Quotient: $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{K}$, provided $K \neq 0$

5. Power: $\lim_{x \to c} [f(x)]^n = L^n$

Strategy for finding limits analytically

1. Direct Substitution
2. Algebraic techniques

(factoring or rationalizing or simplifying)

3. Special Cases

THEOREM 1.9 TWO SPECIAL TRIGONOMETRIC LIMITS

1. $\lim_{x \to 0} \frac{\sin x}{x} = 1$ **2.** $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

Ex 1

$$\lim_{x \to 2} \frac{\sqrt{x+2}}{x-4}$$

Ex 2

$$\lim_{x \to 7} \sec\left(\frac{\pi x}{6}\right)$$

Ex 3

Pythagorean Identities

$$\lim_{x \to 5\pi/3} \cos x$$

Other Trig identities

Ex 4

$$\lim_{x \to 2} \frac{3x^2 - 4x - 4}{2x^2 - 8}$$

$$\lim_{x \to 2} \frac{(3x+2)\cancel{(x-2)}}{2(x+2)\cancel{(x-2)}}$$

$$\frac{8}{8} = 1$$

Ex 5

$$\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 6x + 8}$$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x-4)}$$

$$\frac{12}{-2} = -6$$

Ex 6

$$\lim_{x \to \frac{\pi}{3}} \frac{2\cos^2 x + 3\cos x - 2}{2\cos x - 1}$$

$$\lim_{x \to \frac{\pi}{3}} \frac{\cancel{(2\cos x - 1)}(\cos x + 2)}{\cancel{2\cos x - 1}}$$

$$\frac{1}{2} + 2$$

$$\frac{5}{2}$$

Ex 7

$$\lim_{x \to 0} \frac{\cot x}{\csc x}$$

$$\lim_{x \to 0} \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$$

$$\lim_{x \to 0} \cos x = 1$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Ex 8

$$\lim_{x \to 0} \frac{e^x - e^{2x}}{1 - e^x}$$

Ex 9

$$\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16}$$

Ex 10

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} \cdot \frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)}$$

$$\lim_{x\to 3} \frac{\cancel{x+1-4}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$$

$$\lim_{x\to 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$$

11.) $\lim_{x \to 2} \frac{x-2}{\sqrt{x}-\sqrt{4-x}}$

Ex 12

$$\lim_{x\to 4} \frac{\frac{x}{x+1} - \frac{4}{5}}{x-4} = \lim_{x\to 4} \frac{\frac{5x-4(x+1)}{5(x+1)}}{x-4}$$

$$\lim_{x\to 4} \frac{\frac{x-4}{5(x+1)}}{x-4} = \lim_{x\to 4} \frac{\cancel{x-4}}{5(x+1)} \cdot \frac{1}{\cancel{x-4}} = \frac{1}{25}$$

Ex 13

$$\lim_{x\to 0} \frac{3(1-\cos x)}{x}$$

$$3\left[\lim_{x\to 0} \frac{1-\cos x}{x}\right]$$

$$3\cdot 0$$

$$0$$

Ex 14

$$\lim_{t\to 0} \frac{\sin 3t}{2t}$$

$$\frac{1}{2}\lim_{t\to 0} \frac{3\cdot \sin 3t}{3\cdot t}$$

$$\frac{3}{2}\left[\lim_{t\to 0} \frac{\sin 3t}{3t}\right]$$

$$\frac{3}{2}$$

$$\lim_{x \to 0} \frac{\sin 4x}{11x}$$

Ex 15: Given f(x) = 5x - 2,

$$f(x+h) = 5(x+h) - 2$$

find $\displaystyle\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$.

$$\lim_{h \to 0} \frac{5(x+h) - 2 - (5x - 2)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{5x} + 5h \cancel{- 2} \cancel{- 5x} \cancel{+ 2}}{h}$$

$$5$$

16.) $f(x) = x^2 + 4x$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{(x+h)^2 + 4(x+h) - (x^2+4x)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{4x} + 4h - \cancel{x^2} - \cancel{4x}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x + h + 4)}{\cancel{h}}$$

$2x + 4$

17.) $f(x) = \frac{1}{x}$

Analyze a limit with a table

Analyze a limit with a graph

Analyze a limit analytically

direct substitution, factoring,
rationalizing, simplifying,
special cases

THEOREM 1.8 The Squeeze Theorem

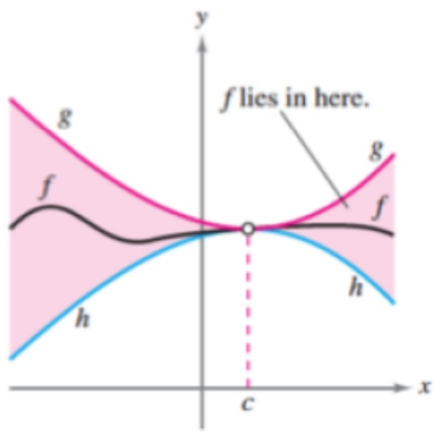
If $h(x) \le f(x) \le g(x)$ for all x in an open interval containing c, except possibly at c itself, and if

$$\lim_{x\to c} h(x) = L = \lim_{x\to c} g(x)$$

then $\lim_{x\to c} f(x)$ exists and is equal to L.

A proof of this theorem is given in Appendix A.

$h(x) \le f(x) \le g(x)$



8.

$$f(x) = \begin{cases} -x^2+3x+3 & \text{for} \quad x<2 \\ 6 & \text{for} \quad x=2 \\ 8-\frac{3}{2}x & \text{for} \quad x>2 \end{cases}$$

$\lim_{x\to 2} f(f(x))$

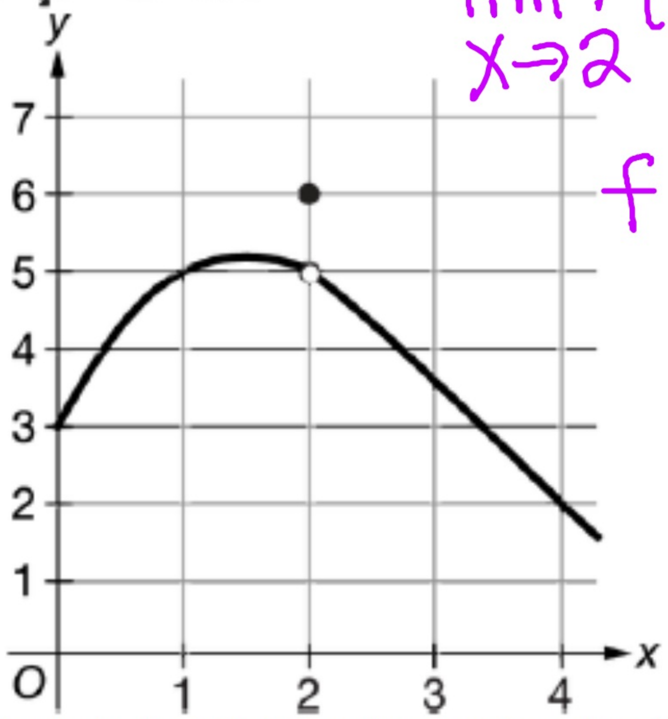
$8-\frac{3}{2}\cdot 5$

$f\left(\lim_{x\to 2} f(x)\right)$

$8-\frac{15}{2}$

$f(5)$

$\frac{1}{2}$



Let f be the piecewise function defined above. Also shown is a portion of the graph of f.
What is the value of $\lim_{x\to 2} f(f(x))$?