

1.3 Finding Limits Analytically

ex: Find the limit analytically. If the limit does not exist, explain.

a) $\lim_{x \rightarrow 2} (5x^2 - 1)$

b) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

c) $\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$

d) $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$

e) $\lim_{x \rightarrow 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$

f) $\lim_{x \rightarrow \frac{3\pi}{2}} \frac{\sin 2x}{\cos x}$

g) $\lim_{x \rightarrow \frac{\pi}{8}} (\sin^2 x + \cos^2 x)$

h) $\lim_{x \rightarrow 0} \sin x \cot x$

i) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x}{1 - \sin x}$

Limit Properties

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \rightarrow c} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow c} g(x) = K$$

1. Scalar multiple: $\lim_{x \rightarrow c} [bf(x)] = bL$
2. Sum or difference: $\lim_{x \rightarrow c} [f(x) \pm g(x)] = L \pm K$
3. Product: $\lim_{x \rightarrow c} [f(x)g(x)] = LK$
4. Quotient: $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{K}, \quad \text{provided } K \neq 0$
5. Power: $\lim_{x \rightarrow c} [f(x)]^n = L^n$

ex: Use the information to evaluate the limits.

$$\lim_{x \rightarrow c} f(x) = 4$$

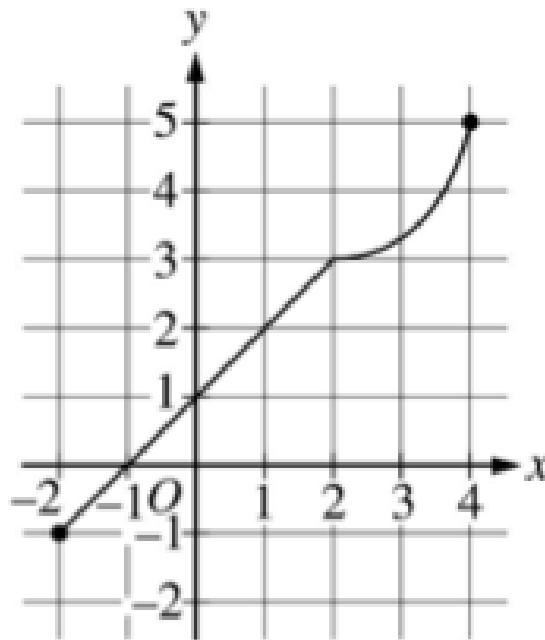
(a) $\lim_{x \rightarrow c} [f(x)]^3$

(b) $\lim_{x \rightarrow c} \sqrt{f(x)}$

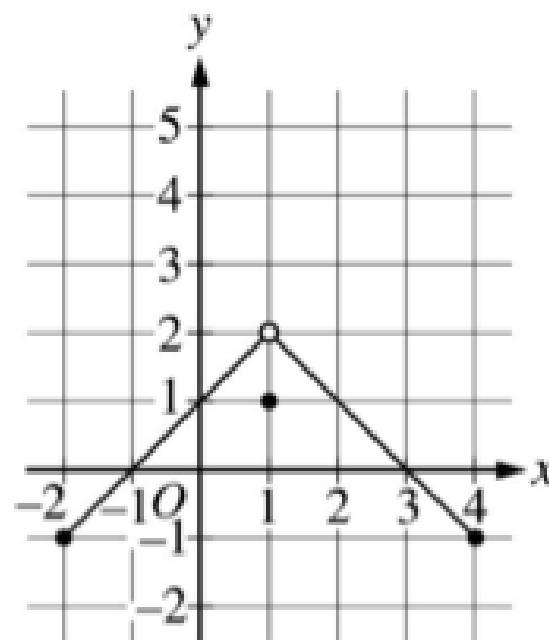
(c) $\lim_{x \rightarrow c} [3f(x)]$

(d) $\lim_{x \rightarrow c} [f(x)]^{3/2}$

ex:



Graph of f



Graph of g

The graphs of the functions f and g are shown above. The value of $\lim_{x \rightarrow 1} f(g(x))$ is

- (A) 1
- (B) 2
- (C) 3
- (D) nonexistent

Special Trigonometric Limits

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

ex: Find the limit analytically. If the limit does not exist, explain.

a) $\lim_{x \rightarrow 0} \frac{\sin 4x}{x}$

b) $\lim_{x \rightarrow 0} \frac{\sin x}{4x}$

c) $\lim_{x \rightarrow 0} \frac{x}{\sin x}$

d) $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x}$

e) $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

f) $\lim_{x \rightarrow 0} \frac{(1 - \cos x)^2}{x}$

$$g) \lim_{x \rightarrow 0} \frac{\sin x}{\sqrt[3]{x}}$$

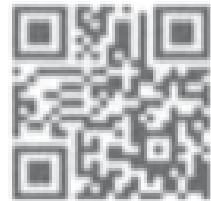
THEOREM 1.8 The Squeeze Theorem

If $h(x) \leq f(x) \leq g(x)$ for all x in an open interval containing c , except possibly at c itself, and if

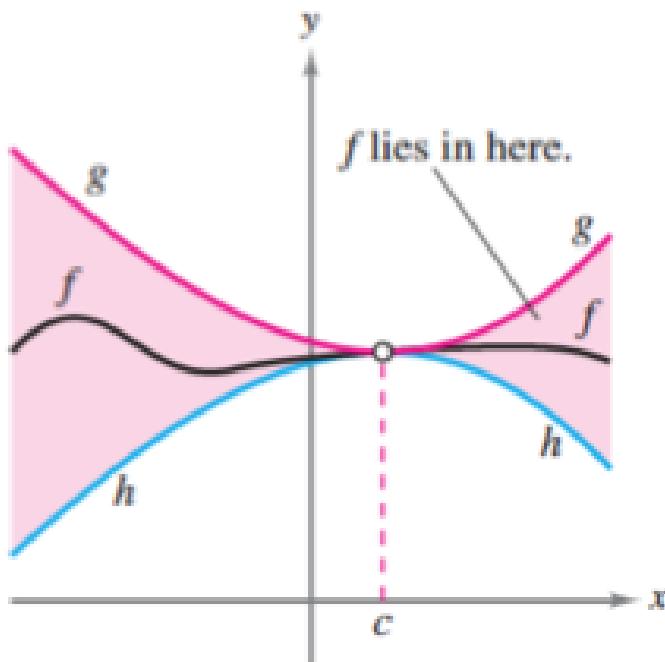
$$\lim_{x \rightarrow c} h(x) = L = \lim_{x \rightarrow c} g(x)$$

then $\lim_{x \rightarrow c} f(x)$ exists and is equal to L .

A proof of this theorem is given in Appendix A.



$$h(x) \leq f(x) \leq g(x)$$



Proof: $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

