1.2 Finding Limits Graphically and Numerically

- Estimate a limit using a numerical or graphical approach.
- Learn different ways that a limit can fail to exist.
- Study and use a formal definition of limit.

1. Numerical approach — Construct a table of values.
2. Graphical approach — Draw a graph by hand or using technology.
3. Analytic approach — Use algebra or calculus.

Ex 1 $\frac{0}{0}$: indeterminate form

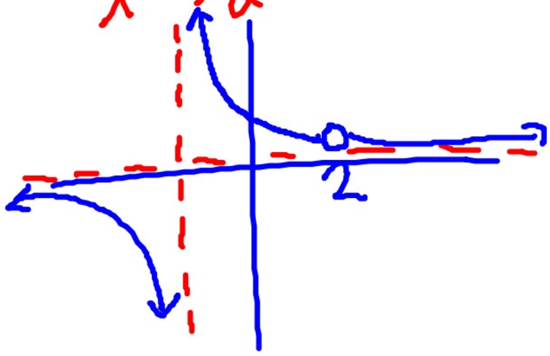
$$\lim_{x\to 2} \frac{x-2}{x^2-4} = .25$$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	.256	.251	.25006	.24994	.24938	.2439

$x \to 2^-$ $\qquad$ $x \to 2^+$

$$\frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)} = \frac{1}{x+2}$$

hole @ $(2, .25)$



Ex 2

$$\lim_{x\to4}\frac{[x/(x+1)]-(4/5)}{x-4} = .04$$

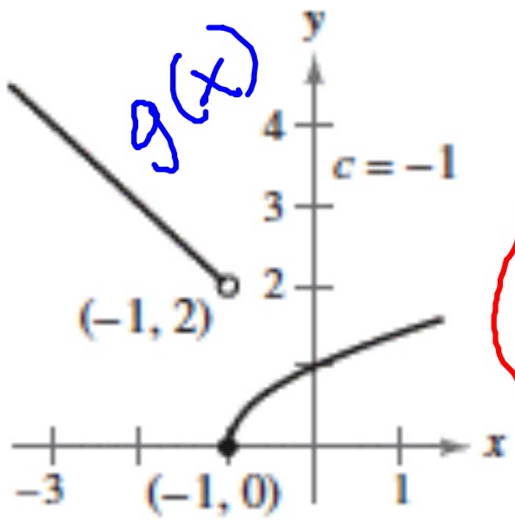
x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	.04082	.04008	.04001	.03999	.03992	.03922

$x \to 4^-$ $x \to 4^+$

COMMON TYPES OF BEHAVIOR ASSOCIATED WITH NONEXISTENCE OF A LIMIT

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

DNE
Type #1



$$\lim_{x \to -1} g(x)$$

Justification

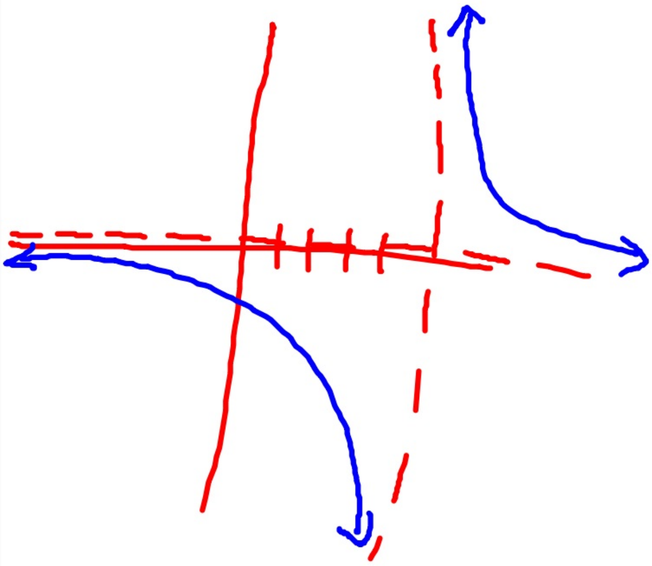
$$\lim_{x \to -1^-} g(x) \neq \lim_{x \to -1^+} g(x)$$

$$2 \neq 0$$

dne

DNE Type #2

2. $\lim_{x \to 5} \frac{2}{x-5}$ dne



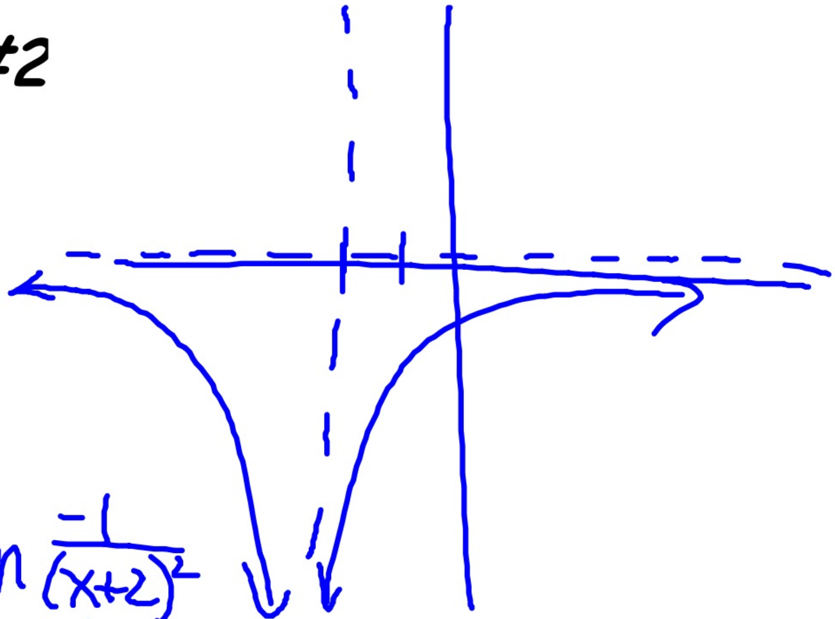
$\lim_{x \to 5^-} \frac{2}{x-5} \neq \lim_{x \to 5^+} \frac{2}{x-5}$

Another type #2

$$\lim_{x \to -2} \frac{-1}{(x+2)^2}$$

$$-\infty$$

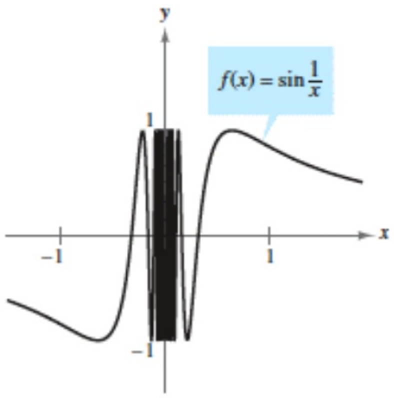
$$\lim_{x \to -2^-} \frac{-1}{(x+2)^2} \neq \lim_{x \to -2^+} \frac{-1}{(x+2)^2}$$



Infinity is a special case of DNE

DNE Type #3

3.



$\lim_{x\to 0} f(x)$ does not exist.

$\lim_{x\to 0} \sin\frac{1}{x}$ dne

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	.544	.506	−.8269	.8268	−.506	−.544

$x \to 0^-$ $x \to 0^+$

(a) $f(-2)$ undefined

(b) $\lim_{x\to -2} f(x)$ dne *Justify

(c) $f(0) = 4$

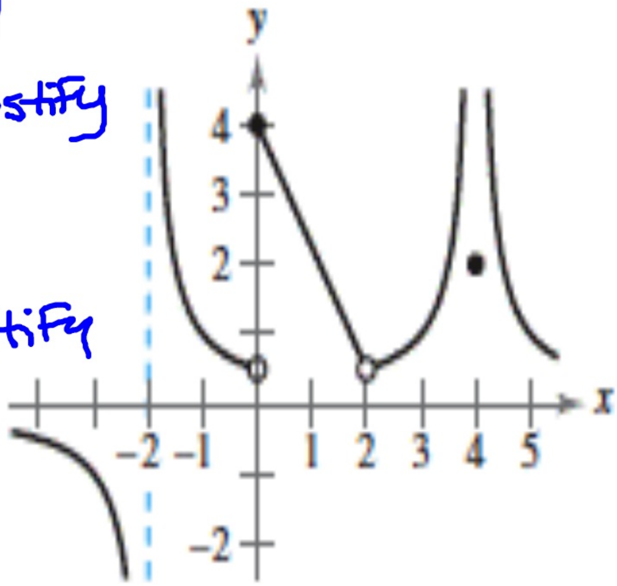
(d) $\lim_{x\to 0} f(x)$ dne *Justify

(e) $f(2)$ undefined

(f) $\lim_{x\to 2} f(x) = \frac{1}{2}$

(g) $f(4) = 2$

(h) $\lim_{x\to 4} f(x)$ ∞ *Justify



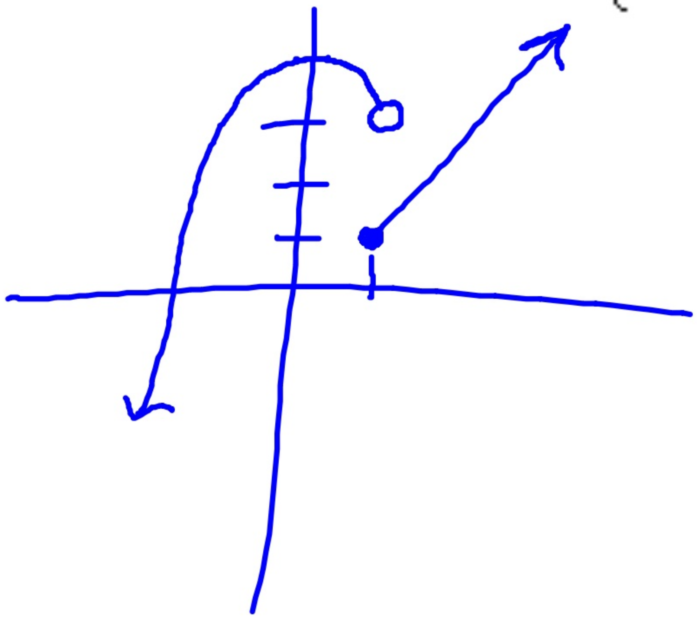
Sketching Piecewise functions

Sketch the graph of the following piecewise function.

$$g(x) = \begin{cases} -x^2 + 4 & \text{if } x < 1 \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$$

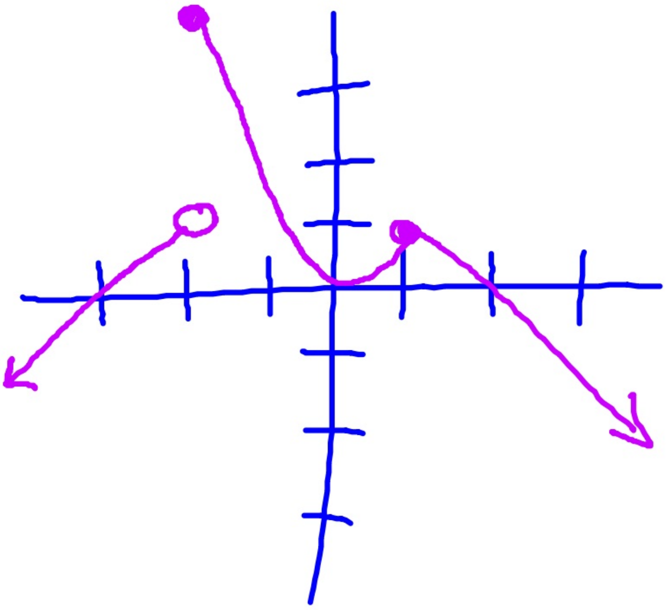
$\lim_{x \to 1} g(x)$

dne



Sketch the graph of the following piecewise function.

$$h(x)=\begin{cases} x+3 & \text{if } x<-2 \\ x^2 & \text{if } -2\le x<1 \\ -x+2 & \text{if } x\ge 1 \end{cases}$$



$\lim_{x\to -2} h(x)$ dne

$\lim_{x\to -2^-} h(x) \neq \lim_{x\to -2^+} h(x)$

$\lim_{x\to 1} h(x) = 1$

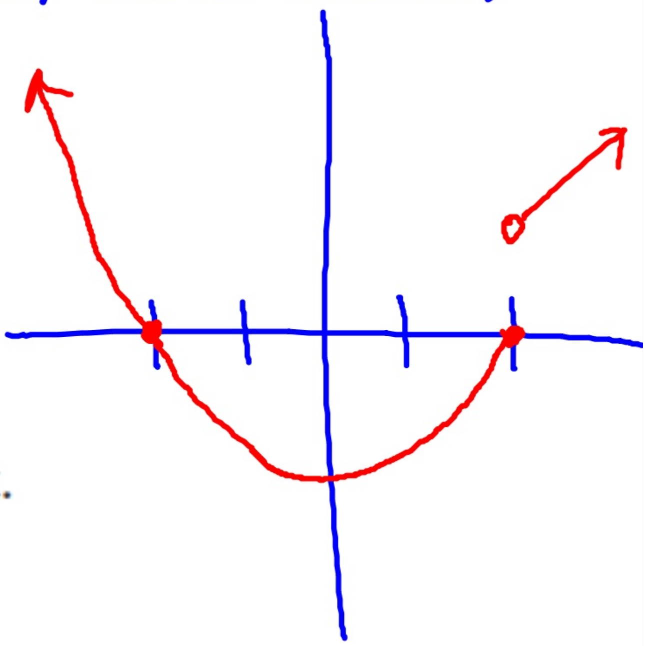
Sketch a graph of a function f that satisfies the given values. (There are many correct answers)

$$f(-2) = 0$$

$$f(2) = 0$$

$$\lim_{x \to -2} f(x) = 0$$

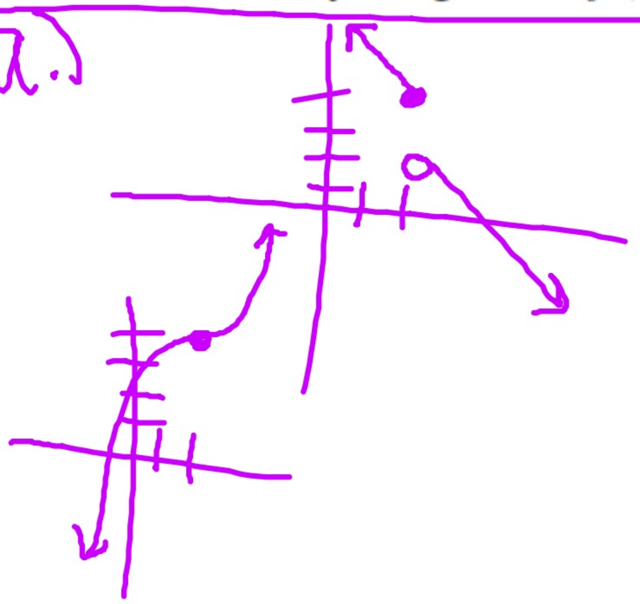
$\lim_{x \to 2} f(x)$ does not exist.



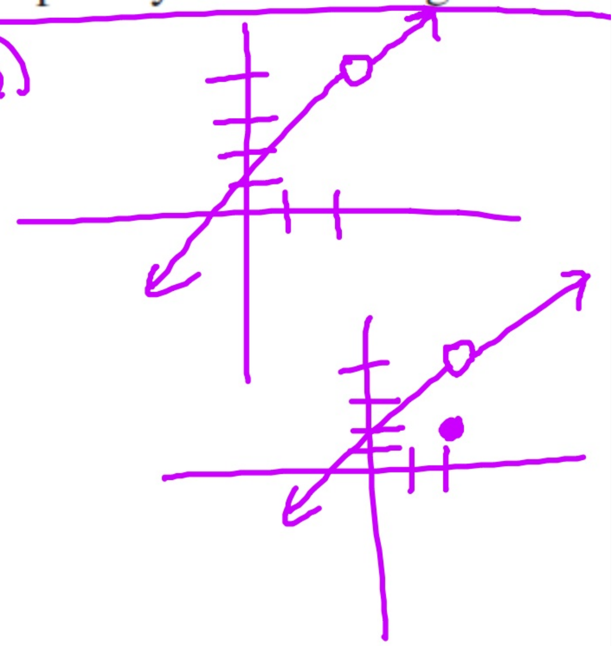
(a) If $f(2) = 4$, can you conclude anything about the limit of $f(x)$ as x approaches 2? Explain your reasoning.

(b) If the limit of $f(x)$ as x approaches 2 is 4, can you conclude anything about $f(2)$? Explain your reasoning.

a.)



b)



a.) $\lim_{x \to 2} \frac{x^2 - x - 7}{x - 4}$

$\frac{-5}{-2}$

$\frac{5}{2}$

b.) $\lim_{x \to 5} \sin \frac{\pi x}{3}$

$\sin \frac{5\pi}{3}$

$-\frac{\sqrt{3}}{2}$

$$\frac{\left(\frac{1}{3+x} - \frac{1}{3}\right)}{\left(\frac{x}{1}\right)} \cdot \frac{3(3+x)}{3(3+x)}$$

$$\frac{3-(3+x)}{3x(3+x)}$$

$$\frac{-\cancel{x}}{3x(3+x)} = \frac{-1}{3(3+x)}$$