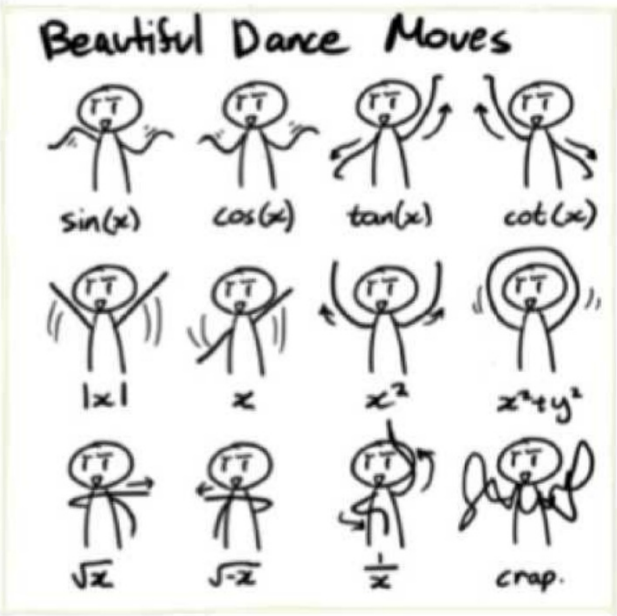
Library of Functions & Transformations - Day 1



Parent Functions (Mother Functions)

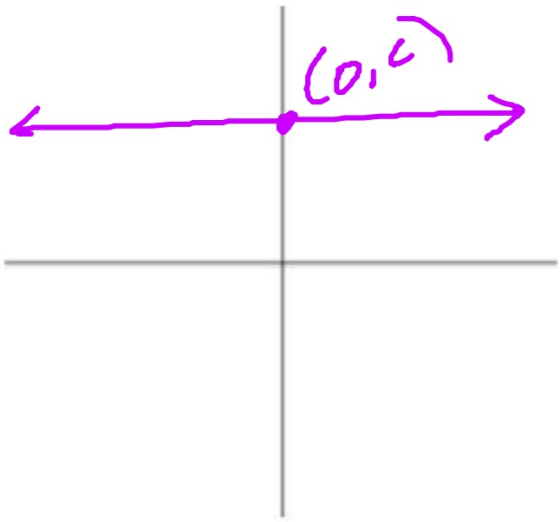
A parent function is the simplest function of a family of functions. For the family of quadratic functions,

$y = ax^2 + bx + c$, the simplest function. of this form is $y = x^2$.

In this lesson we will examine several "families" of functions. A family of functions is a set of functions whose equations have a similar form. The "parent" of the family is the equation in the family with the simplest form. For example, $y = x^2$ is a parent to other functions, such as $y = 2x^2 - 5x + 3$.

1. Constant

$f(x) =$ C



Set

D: $\{x \mid x \in R\}$

R: $\{y \mid y = c\}$

Interval

D: $(-\infty, \infty)$

R: $[c]$

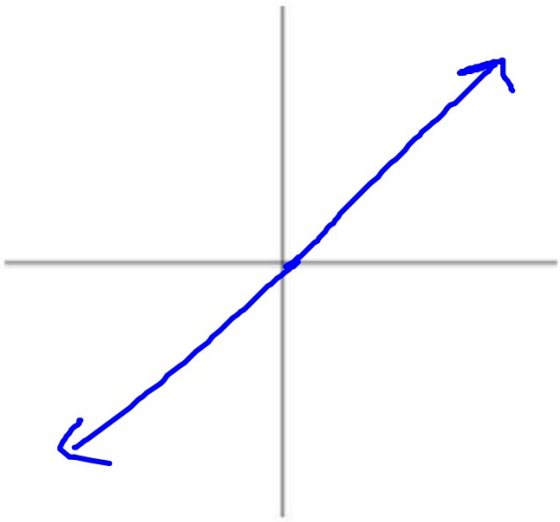
End Behavior

$x \to -\infty \quad f(x) \to$ c

$x \to \infty \quad f(x) \to$ c

2. Identity

$f(x) =$ X



Set

D: $\{x \mid x \in R\}$

R: $\{y \mid y \in R\}$

Interval

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

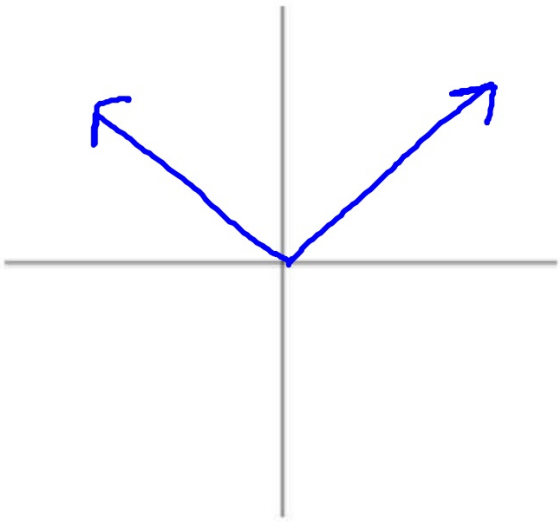
End Behavior

$x \to -\infty \quad f(x) \to -\infty$

$x \to \infty \quad f(x) \to \infty$

3. Absolute Value

$f(x) =$ $|x|$



Set

D: $\{x \mid x \in R\}$

R: $\{y \mid y \geq 0\}$

Interval

D: $(-\infty, \infty)$

R: $[0, \infty)$

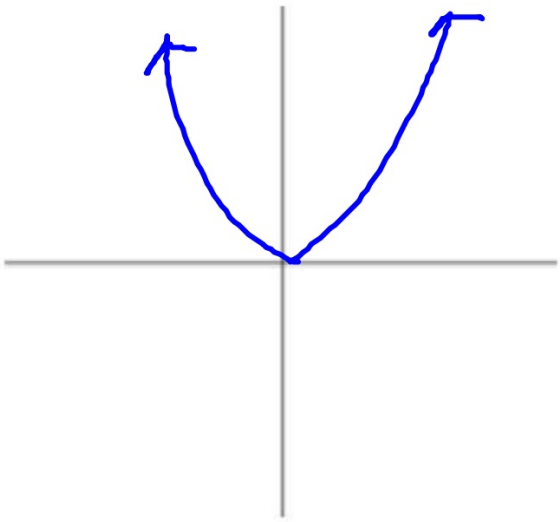
End Behavior

$x \to -\infty \quad f(x) \to \infty$

$x \to \infty \quad f(x) \to \infty$

4. Quadratic

$f(x) =$ x^2



Set

D: $\{x \mid x \in R\}$

R: $\{y \mid y \geq 0\}$

Interval

D: $(-\infty, \infty)$

R: $[0, \infty)$

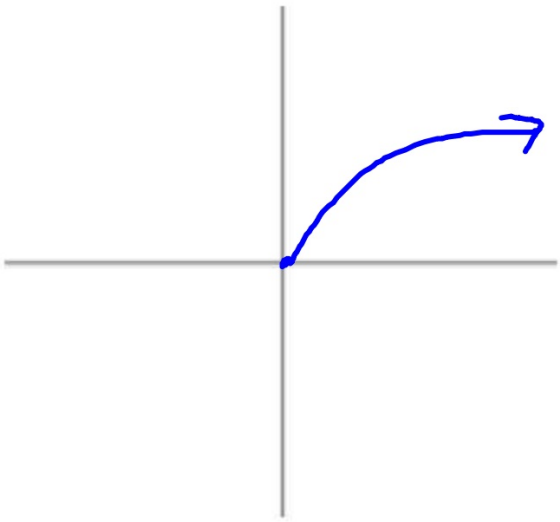
End Behavior

$x \to -\infty \quad f(x) \to \infty$

$x \to \infty \quad f(x) \to \infty$

5. Square Root

$f(x) =$ $\sqrt{x}$



Set

D: $\{x \mid x \geq 0\}$

R: $\{y \mid y \geq 0\}$

Interval

D: $[0, \infty)$

R: $[0, \infty)$

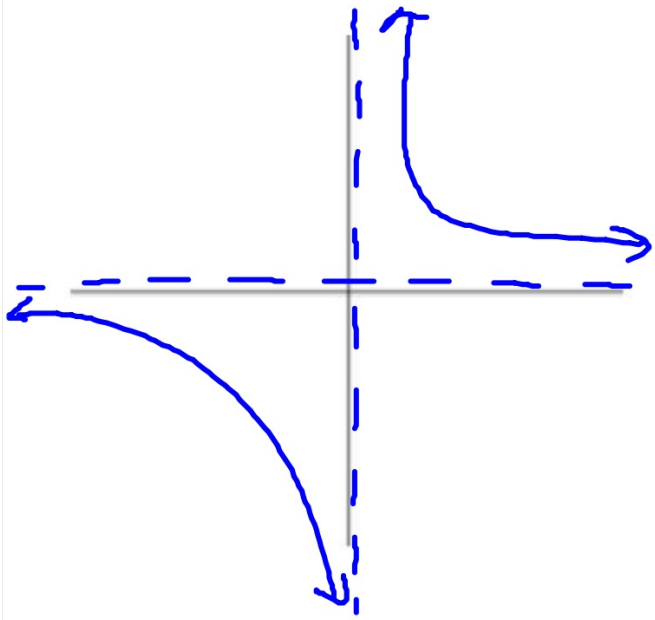
End Behavior

$x \to -\infty \quad f(x) \to$ N/A

$x \to \infty \quad f(x) \to \infty$

6. Reciprocal

$$f(x) = \frac{1}{x}$$



Set

D: $\{x \mid x \neq 0\}$

R: $\{y \mid y \neq 0\}$

Interval

D: $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

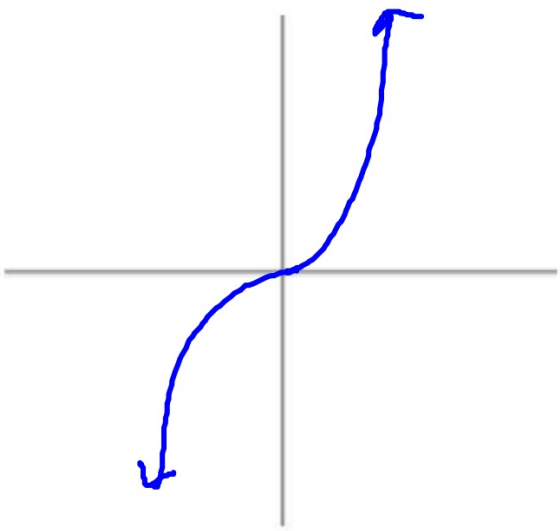
End Behavior

$x \to -\infty \quad f(x) \to 0$

$x \to \infty \quad f(x) \to 0$

7. Cubic

$f(x) =$ x^3



Set

D: $\{x \mid x \in R\}$

R: $\{y \mid y \in R\}$

Interval

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

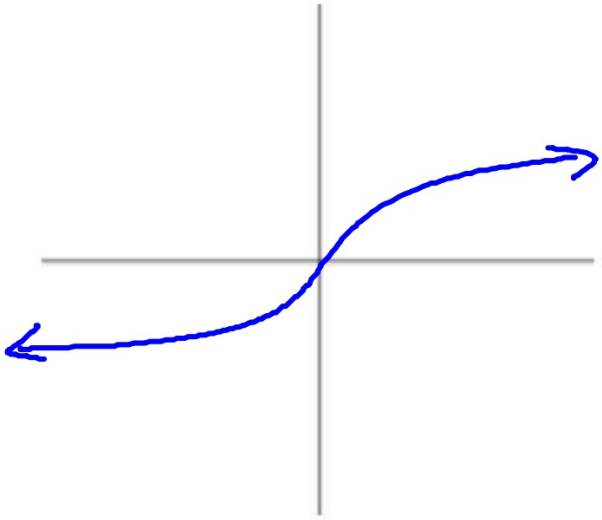
End Behavior

$x \to -\infty \quad f(x) \to -\infty$

$x \to \infty \quad f(x) \to \infty$

8. Cube Root

$f(x) =$ $\sqrt[3]{x}$



Set

D: $\{x \mid x \in R\}$

R: $\{y \mid y \in R\}$

Interval

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

End Behavior

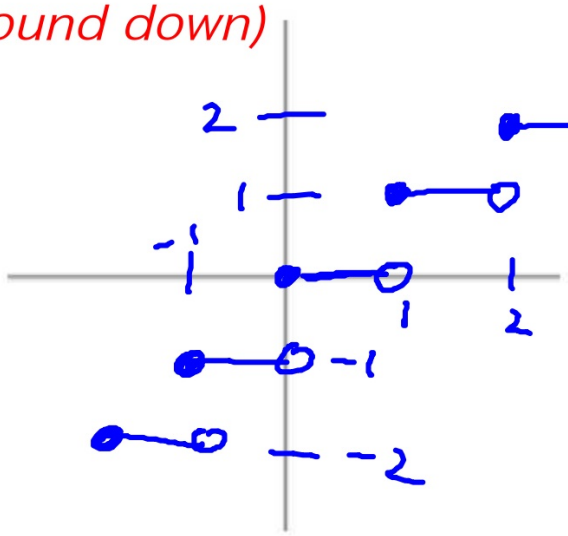
$x \to -\infty \quad f(x) \to -\infty$

$x \to \infty \quad f(x) \to \infty$

9. Greatest Integer

Greatest Integer less than or equal to the value (round down)

$f(x) =$ [x] or ⌊x⌋



Set

D: $\{x \mid x \in R\}$

R: $\{y \mid y \in Z\}$

Interval

D: $(-\infty, \infty)$

R: N/A

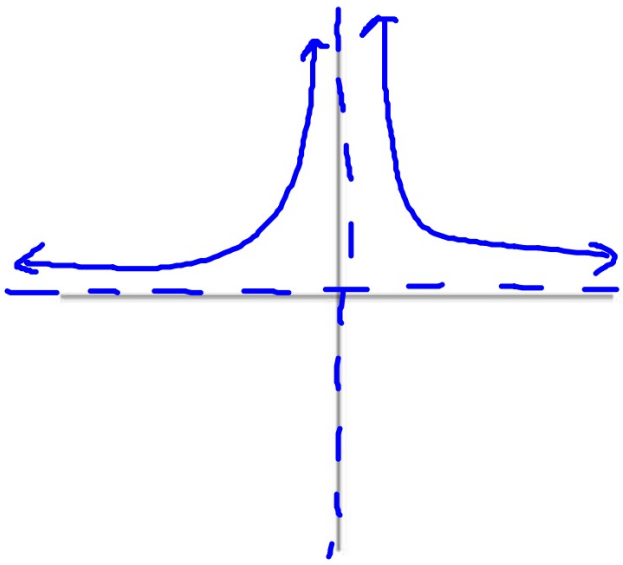
End Behavior

$x \to -\infty \quad f(x) \to -\infty$

$x \to \infty \quad f(x) \to \infty$

10. Reciprocal of a Square

$f(x) =$ $\frac{1}{x^2}$



Set

D: $\{x \mid x \neq 0\}$

R: $\{y \mid y > 0\}$

Interval

D: $(-\infty, 0) \cup (0, \infty)$

R: $(0, \infty)$

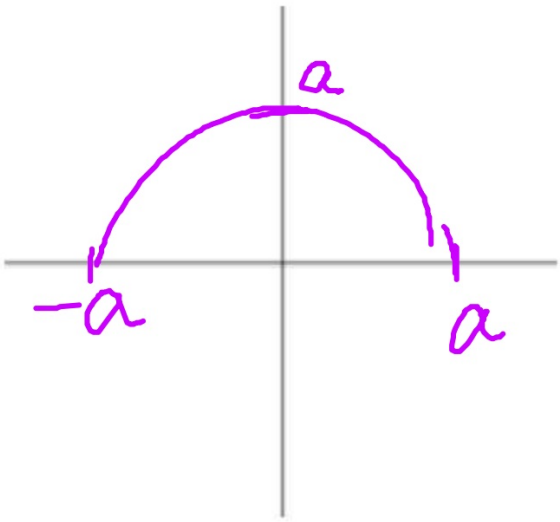
End Behavior

$x \to -\infty \quad f(x) \to 0$

$x \to \infty \quad f(x) \to 0$

11. Semicircle

$y = \sqrt{9 - x^2}$ $\quad f(x) = \sqrt{a^2 - x^2}$



Set

D: $\{x \mid -a \le x \le a\}$

R: $\{y \mid 0 \le y \le a\}$

Interval

D: $[-a, a]$

R: $[0, a]$

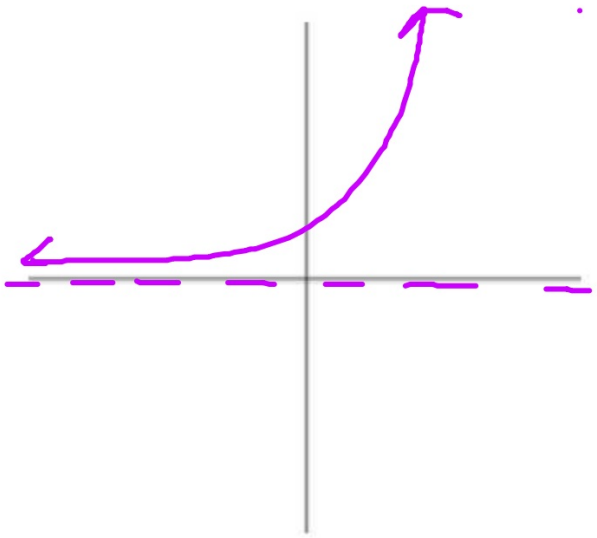
End Behavior

$x \to -\infty \quad f(x) \to$ N/A

$x \to \infty \quad f(x) \to$ N/A

12. Exponential Growth

$f(x) =$ $a \cdot b^x$ $\quad b > 1$



Set

D: $\{x \mid x \in R\}$

R: $\{y \mid y > 0\}$

Interval

D: $(-\infty, \infty)$

R: $(0, \infty)$

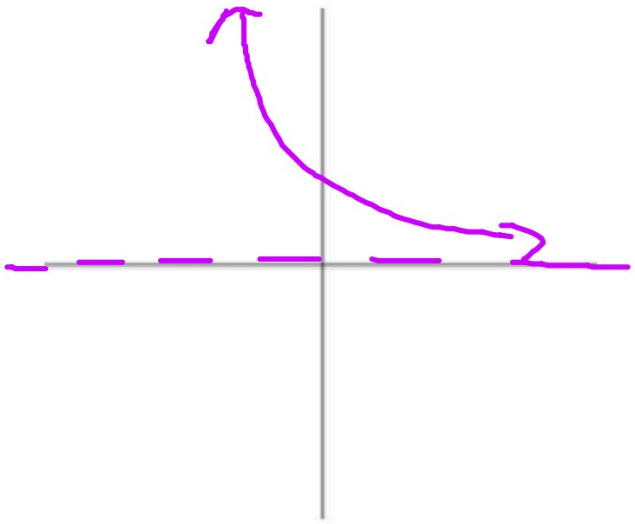
End Behavior

$x \to -\infty \quad f(x) \to 0$

$x \to \infty \quad f(x) \to \infty$

13. Exponential Decay

$f(x) =$ $a \cdot b^x$ $0 < b < 1$



Set

D: $\{x \mid x \in R\}$

R: $\{y \mid y > 0\}$

Interval

D: $(-\infty, \infty)$

R: $(0, \infty)$

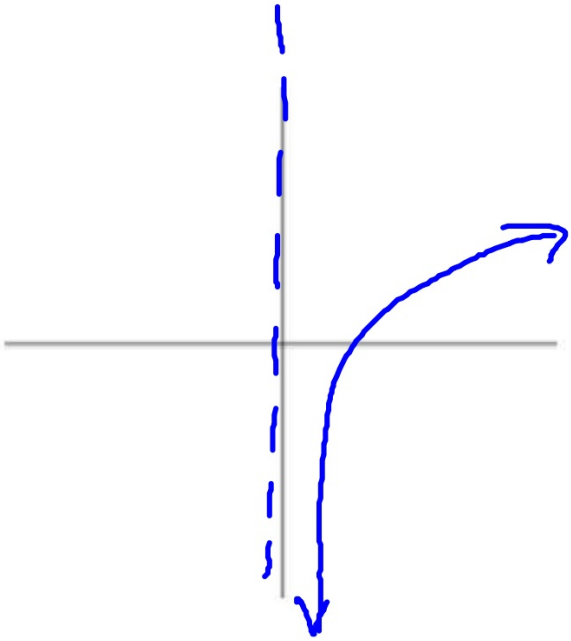
End Behavior

$x \to -\infty$ $f(x) \to$ ∞

$x \to \infty$ $f(x) \to$ 0

14. Natural Logarithm

$f(x) =$ ln x



Set

D: $\{x \mid x > 0\}$

R: $\{y \mid y \in R\}$

Interval

D: $(0, \infty)$

R: $(-\infty, \infty)$

End Behavior

$x \to -\infty \quad f(x) \to$ N/A

$x \to \infty \quad f(x) \to \infty$

Function Transformations

$$y = af\left(b\left(x-h\right)\right)+k$$

$y = 7(x-4)^2 + 6$

Types of Transformations

- Shifts (vertical and horizontal)
- Dilations (vertical and horizontal)
- Reflections (about the x-axis, y-axis and origin)

Shifts

$$y = af\left(b\left(x-h\right)\right)+k$$

Vertical Shifts

Consider: k

k>o up

k<o down

Horizontal Shifts

Consider: h

h>o right

h<o left

$(x-\frac{1}{2})$

$(x--2)$

$(x+2)$

ex: Describe the transformations.

$$f(x) = |x + 7| + 4$$

$|x - -7|$

$h = -7$: left 7 units

$k = 4$: up 4 units

Dilations

$$y = af\left(b\left(x-h\right)\right)+k$$

Vertical

Consider: a

$|a| > 1$ stretch by a

$|a| < 1$ shrink by a

Horizontal

Consider: b

$|b| > 1$ shrink by $\left|\frac{1}{b}\right|$

$|b| < 1$ stretch by $\left|\frac{1}{b}\right|$

ex: Describe the transformations.

$$f(x)=\frac{1}{3}\sqrt{2x-5}+1$$

$$f(x)=\frac{1}{3}\sqrt{2\left(x-\frac{5}{2}\right)}+1$$

right 5/2 units

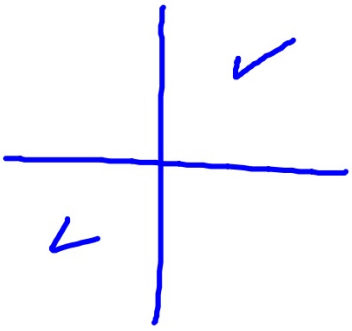
up 1 unit

vertical shrink by a factor of 1/3

horizontal shrink by a factor of ~~2~~ 1/2

Reflections

$$y = af\left(b\left(x-h\right)\right)+k$$



<u>About the x-axis</u>

a<o

<u>About the y-axis</u>

b<o

<u>About the origin</u>

a<o & b<o

ex: Describe the transformations.

a) $y = -3(x+1)^2$

left one unit

vertical stretch by a factor of 3

reflection over x-axis

b) $y = \frac{1}{2}[3-2x]+1$

$y = \frac{1}{2}\left[-2\left(x-\frac{3}{2}\right)\right]+1$

reflection on y-axis

vertical shrink by 1/2

horizontal shrink by 1/2

right 3/2 units

up 1 unit

c) $y = 4-\sqrt{-5x+10}$

$y = -\sqrt{-5(x-2)}+4$

reflection over origin

horizontal shrink by 1/5

right 2 units

up 4 units

Sketching Graphs with "Key Points"

- Absolute Value
- Quadratic
- Square Root
- Cubic
- Cube Root

Process

1. Plot the key point.
2. Make a table of values.

ex: Sketch and state the D/R.

a) $y = -\sqrt[3]{x-4}+1$

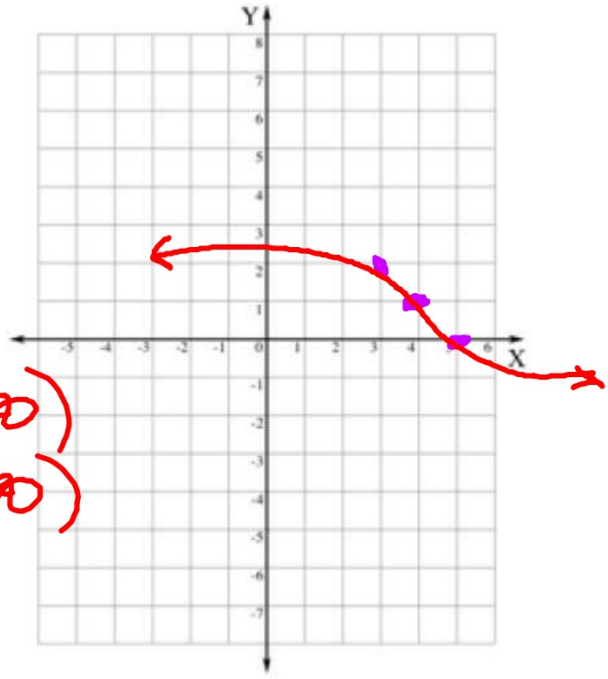
key point: (4, 1)

reflect over x-axis

x	y
5	0
3	2

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$



Domain:	Range:

ex: Sketch and state the D/R.

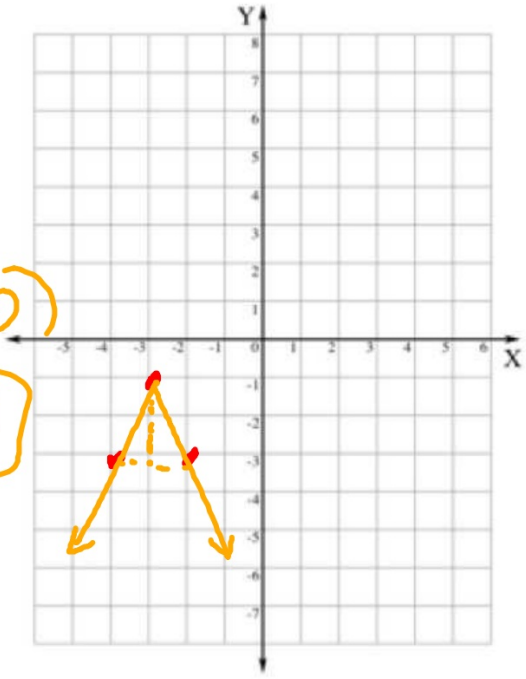
b) $f(x) = -2|x+3|-1$

Key pt $(-3,-1)$

x	y
-4	-3
-2	-3

D: $(-\infty, \infty)$

R: $(-\infty, -1]$



Domain:	Range:

ex: Sketch and state the D/R.

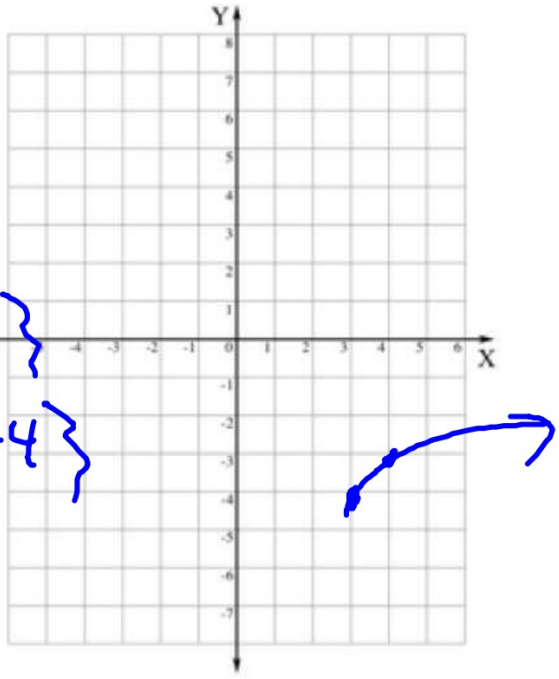
c) $f(x) = \sqrt{x-3} - 4$

(3, -4)

x	y
4	-3

D: $\{x \mid x \geq 3\}$

R: $\{y \mid y \geq -4\}$



Domain:	Range:

ex: Sketch and state the D/R.

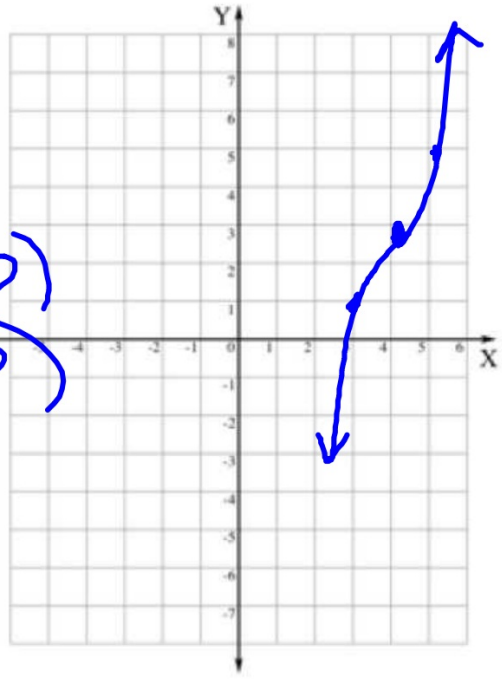
d) $y = 2(x-4)^3 + 3$

(4, 3)

x	y
3	1
5	5

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$



Domain:	Range:

ex: Sketch and state the D/R.

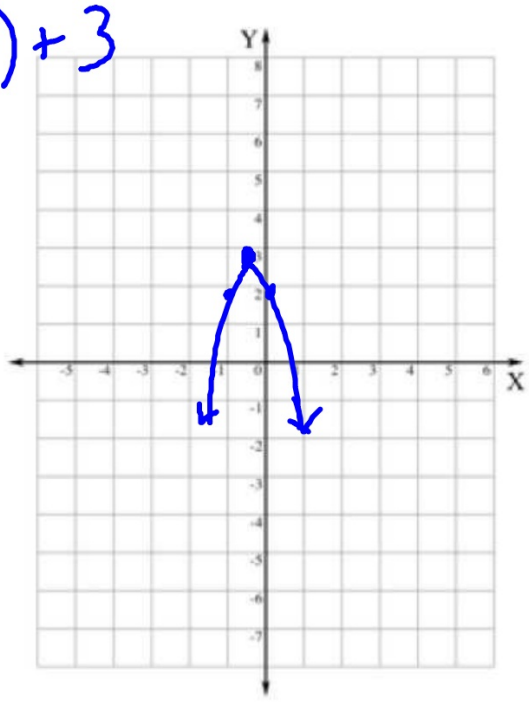
e) $f(x) = 3 - (2x+1)^2 = -(2x+1)^2 + 3$

$\left(-\frac{1}{2}, 3\right)$

reflect over x-axis

D: $(-\infty, \infty)$

R: $(-\infty, 3]$



Domain:	Range: