transformation of functions

$$y = a\,f(b(x-h)) + K$$

Shifts

- vertical
  - up $K > 0$
  - down $K < 0$
- horizontal
  - right $h > 0$
  - left $h < 0$

Dilations

- Vertical
  - stretch $|a|>1$
  - shrink $0<|a|<1$
- Horizontal
  - stretch $0<|b|<1$
  - shrink $|b|>1$

Reflections

- x-axis $a < 0$
- y-axis $b < 0$

Describe the transformations

① $y = 3(x-4)^2 + 5$

$\rightarrow 4 \quad \uparrow 5$

vertical stretch

$y = a(x-h)^2 + k$

vertex : (h, k)

② $y = -|2x+6| - 1$

rewrite

$y = -|2(x+3)| - 1$

- $\leftarrow 3 \quad \downarrow 1$
- reflect. wl x-axis
- horizontal shrink

③ $y = \frac{1}{2}[5 - x] + 6$

rewrite

$y = \frac{1}{2}[-1(x-5)] + 6$

$\rightarrow 5$

$\uparrow 6$

vertical shrink

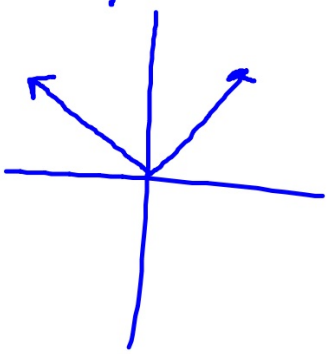
reflection over y-axis

Sketch:

Graphs with a "key point" (h, k)

absolute value
square root
cube root
quadratic
cubic

$y = |x|$



4 $y = -|3x+9| - 4$

rewrite

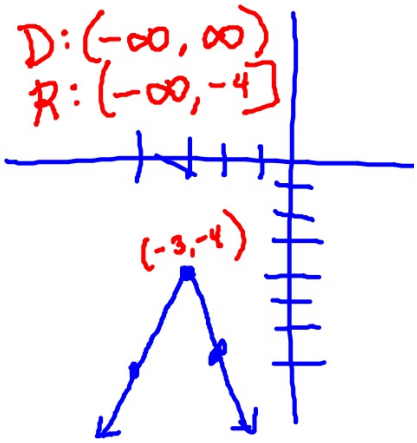
$y = -|3(x+3)| - 4$

Key point $(-3, -4)$

horiz. shrink

x	y
-2	-7
-3	-4
-4	-7

D: $(-\infty, \infty)$
R: $(-\infty, -4]$

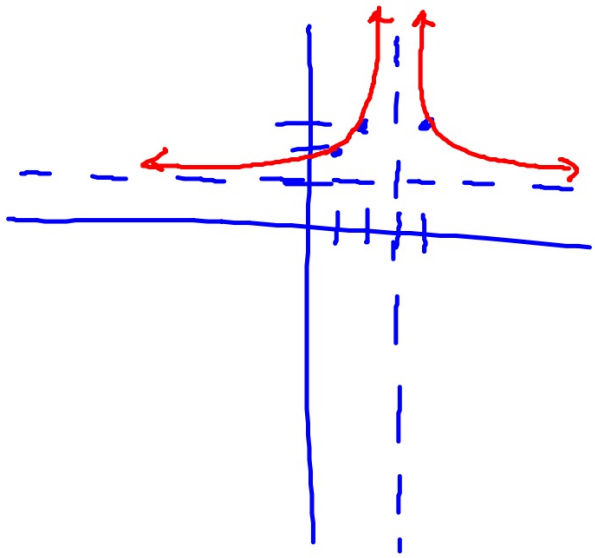


Sketch
Graphs with asymptotes

natural log
exp. growth/decay
reciprocal
reciprocal of a square

5 $y = \frac{2}{(x-3)^2} + 1$

VA: $x = 3$ HA: $y = 1$



D: $\{x \mid x \neq 3\}$

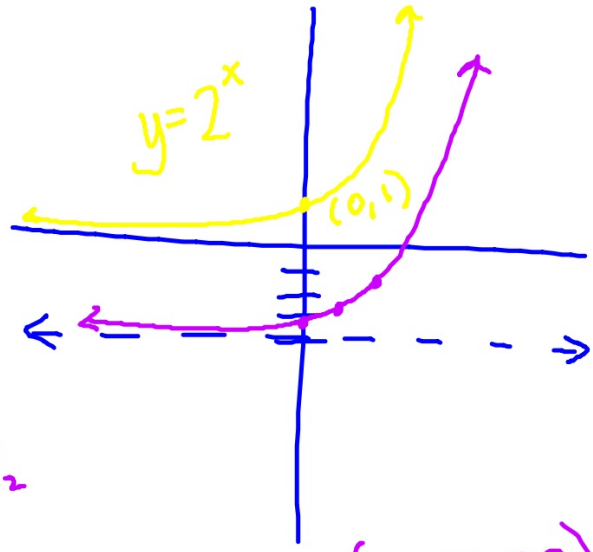
R: $\{y \mid y > 1\}$

x	y
4	3
2	3
1	1.5

6 $y = 2^{x-1} - 4$

exp. growth

HA : $y = -4$



x	y
0	$-3\frac{1}{2}$
1	-3
2	-2

D: $(-\infty, \infty)$

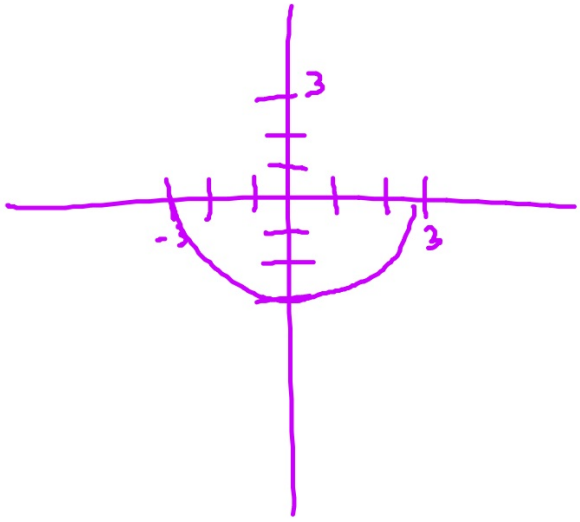
R: $(-4, \infty)$

7 $y = -\sqrt{9 - x^2}$

Center: $(0, 0)$
radius: 3
reflect w/ x-axis
$(a = -1)$

Semicircle

Center (h, k)
Radius

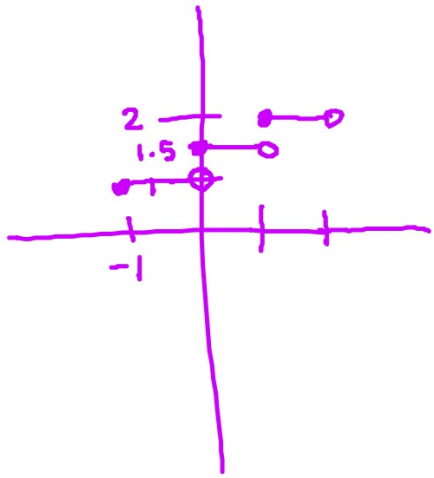


Greatest Integer

Key point (h, k)
Bar length: 1/b
Distance between the bars: a

skip range

8 $y = \frac{1}{2}[x-1]+2$



Key point
(1, 2)

dist. between bars
$\frac{1}{2}$

b=1
bar length: 1

(9) $y = 2\left[\frac{1}{3}x\right]$

Do #1-12

Key point: $(0,0)$

bar length: $\frac{1}{\frac{1}{3}} = 3$

distance: 2

vert. stretch

horiz. stretch