Transformation of functions

$$y = a\,f(b(x-h)) + k$$

$\boxed{\text{Shifts}}$

- vertical
  - up $k > 0$
  - down $k < 0$
- horizontal
  - right $h > 0$
  - left $h < 0$

$y = (x+1)^2$ left 1

$y = (x-1)^2$ right 1

Dilations

- Vertical
  - stretch $|a| > 1$
  - shrink $0 < |a| < 1$

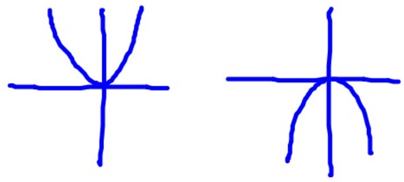
$$\boxed{y = af(b(x-h)) + k}$$

- Horizontal
  - stretch $0 < |b| < 1$
  - shrink $|b| > 1$

Reflections

- x-axis $a < 0$
- y-axis $b < 0$

$y = x^2$ $y = -x^2$



Describe the transformations

① $y = 3(x-4)^2 + 5$ parent: $y = x^2$

$\rightarrow 4$

$\uparrow 5$

vertical stretch

② $y = -|2x+6| - 1$ parent: $y = |x|$

rewrite $-|2(x+3)| - 1$

reflection with x-axis

$\leftarrow 3$ $\downarrow 1$

horizontal shrink

3. $y = \frac{1}{2}[\![5 - x]\!] + 6$

rewrite: $\frac{1}{2}[\![-1(x-5)]\!] + 6$

$\rightarrow 5$

$\uparrow 6$

vertical shrink

reflection with y-axis

Sketch:

Graphs with a "key point" (h, k)

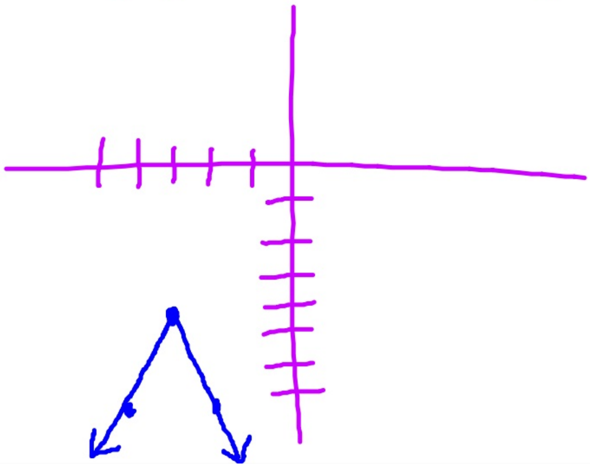
absolute value
square root
cube root
quadratic
cubic

4 $y = -|3x+9| - 4$

$y = -|3(x+3)| - 4$

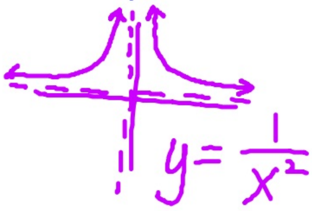
Key point: $(-3, -4)$

x	y
-2	-7
-3	-4
-4	-7



Sketch
Graphs with asymptotes

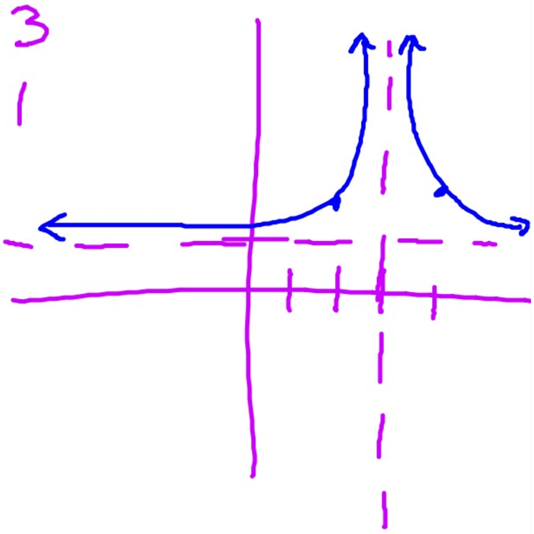
natural log
exp. growth/decay
reciprocal
reciprocal of a square



⑤ $y=\frac{2}{(x-3)^2}+1$

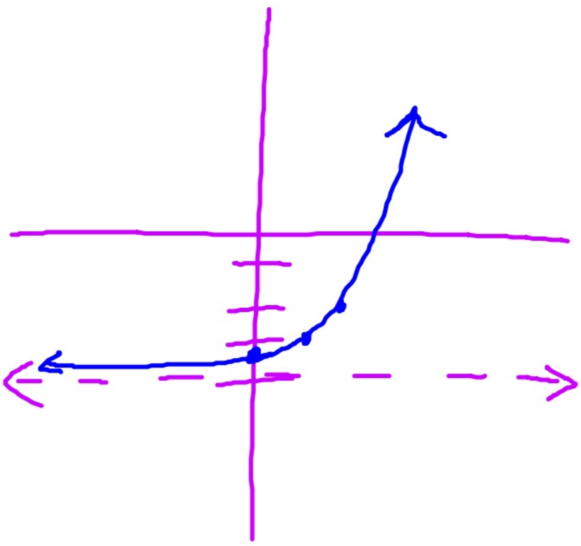
VA: $x=3$
HA: $y=1$

x	y
4	3
2	3



⑥ $y = 2^{x-1} - 4$

x	y
0	-3.5
1	-3
2	-2

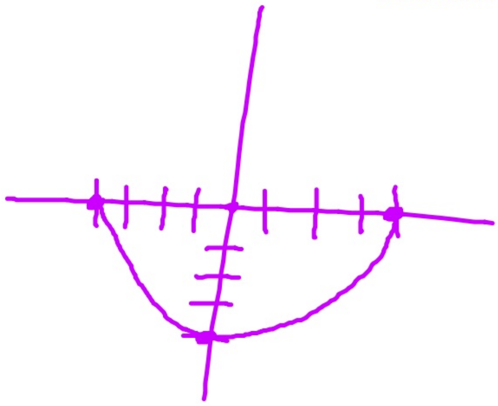


7 $y = -\sqrt{16 - x^2}$

Semicircle

Center (h, k)
Radius

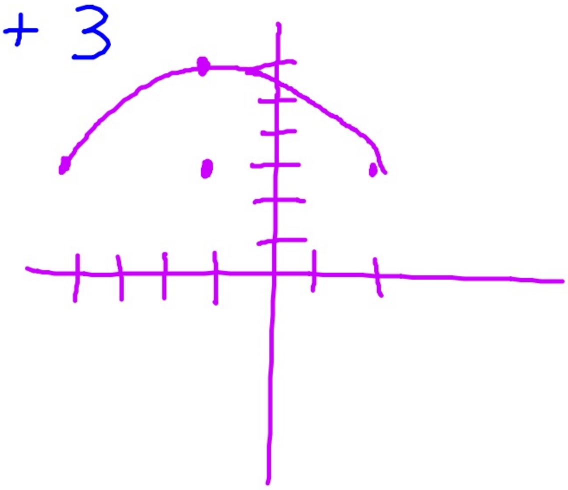
Center: (0, 0)
radius: 4



⑧ $y = \sqrt{9 - (x+1)^2} + 3$

center: $(-1, 3)$

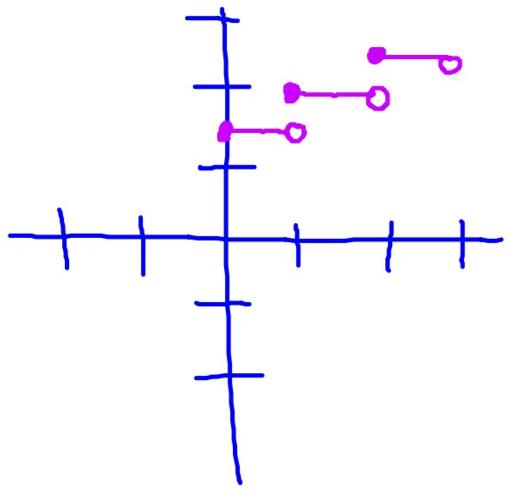
radius: 3



Greatest Integer

Key point (h, k)
Bar length: 1/b
Distance between the bars: a

9 $y = \frac{1}{2}[x-1] + 2$



Key point : (1, 2)
bar length : $\frac{1}{1} = 1$
distance = $\frac{1}{2}$

(10) $y = 3\left[\frac{x}{2}\right] = 3\left[\frac{1}{2}x\right]$

Key point: $(0,0)$

bar length: $\frac{1}{\frac{1}{2}} = 2$

distance: $a = 3$